

MARKET RISK IN COMMODITY MARKETS: A VaR APPROACH

Pierre Giot^{1,3} and Sébastien Laurent²

November 2002

Abstract

We put forward Value-at-Risk models relevant for commodity traders who have long and short trading positions in commodity markets. In a five-year out-of-sample study on aluminium, copper, nickel, Brent crude oil and WTI crude oil daily cash prices and cocoa nearby futures contracts, we assess the performance of the RiskMetrics, skewed Student APARCH and skewed student ARCH models. While the skewed Student APARCH model performs best in all cases, the skewed Student ARCH model delivers good results and its estimation does not require non-linear optimization procedures. As such this new model could be relatively easily integrated in a spreadsheet-like environment and used by market practitioners.

Keywords: Value-at-Risk, skewed Student distribution, ARCH, APARCH, commodity markets

JEL classification: C52, C53, G15

¹Department of Business Administration & CEREFIM at University of Namur, Rempart de la Vierge, 8, 5000 Namur, Belgium, Phone: +32 (0) 81 724887, Email: pierre.giot@fundp.ac.be, and Center for Operations Research and Econometrics (CORE) at Université catholique de Louvain, Belgium

²Centre de Recherche en Economie et Statistique (CREST), 15 Bld. G. Péry, 92245 Malakoff, France, Email: sebastien.laurent@ensae.fr, and CORE at Université catholique de Louvain, Belgium

³Corresponding author

1 Introduction

Managing and assessing risk is a key issue for financial institutions. The 1988 Basel Accord set guidelines for credit and market risk, enforcing the 8% rule or Cooke ratio. Regarding market risk, the total capital requirement for a financial institution is defined as the sum of the requirements for positions in equities, interest rates, foreign exchange and gold and commodities. This sum is a major determinant of the eligible capital of the financial institution based on the 8% rule. Because of this rather arbitrary 8% rule (which originates from credit risk) and the fact that diversification is not rewarded (computing the sum of the parts assumes a correlation of 1 across assets), the 1988 rules were much criticized by market participants and led to the introduction of the 1996 Amendment for computing market risk. This framework suggests an alternative approach as to how the market risk capital requirement should be computed, allowing the use of an internal model to compute the maximum loss over 10 trading days at a 99% confidence level. This set the stage for the so-called Value-at-Risk models, where a VaR model can be broadly defined as a quantitative tool whose goal is to assess the possible loss that can be incurred by a financial institution over a given time period and for a given portfolio of assets: “in the context of market risk, VaR measures the market value exposure of a financial instrument in case tomorrow is a statistically defined bad day” (Saunders and Allen, 2002). VaR’s popularity and widespread use in financial institutions stem from its easy-to-understand definition and the fact that it aggregates the likely loss of a portfolio of assets into one number expressed in percent or in a nominal amount in the chosen currency. Next to the regulatory framework, VaR models are also used to quantify the risk/return profile of active market participants such as traders or asset managers. Further general information about VaR techniques and regulation issues are available in Dowd (1998), Jorion (2000) or Saunders (2000). Most studies in the VaR literature focus on the computation of the VaR for financial assets such as stocks or bonds, and they usually deal with the modelling of VaR for negative returns.¹ Recent examples are the books by Dowd (1998), Jorion (2000) or the papers by van den Goorbergh and Vlaar (1999), Danielsson and de Vries (2000), Vlaar (2000) or Giot and Laurent (2002).

In this paper we address the computation of the VaR for long and short trading positions in commodity markets. Quite interestingly, few papers deal with commodity markets and market risk management in this framework. Some recent work on the modelling of volatility and VaR in commodity markets include Kroner, Kneafsey, and Claessens (1994) and Manfredo and Leuthold (1998). Thus we model VaR for commodity traders having either bought the commodity (long position) or short-sold it (short position).² In the first case, the risk comes from a drop in the

¹Indeed, it is assumed that traders or portfolio managers have long trading positions, i.e. they bought the traded asset and are concerned when the price of the asset falls.

²An asset is short-sold by a trader when it is first borrowed and subsequently sold on the market. By doing this,

price of the commodity, while the trader loses money when the price increases in the second case (because he would have to buy back the commodity at a higher price than the one he got when he sold it). Correspondingly, one focuses in the first case on the left side of the distribution of returns, and on the right side of the distribution in the second case. Note that this type of VaR modelling could be undertaken with a non-parametric model that would first model the quantile in the left tail of the distribution of returns, and then deal with the right tail. Our approach is however a pure parametric one, where we consider models that jointly deliver accurate VaR forecasts, i.e. both for the left and right tails of the distribution of returns. We first use the skewed Student APARCH model of Lambert and Laurent (2001) and show that this model accurately forecasts the one-day-ahead VaRs for long and short positions in commodity markets. The empirical application focuses on aluminium, copper, nickel, Brent crude oil and WTI crude oil daily cash prices, and cocoa nearby futures contracts. Because the RiskMetrics method is widely used by market practitioners, we also compute the relevant VaR measures in this framework. Not surprisingly however, and because the distribution of returns is leptokurtic and (in some cases) skewed, the RiskMetrics method often does not deliver good results. In a second step, we introduce the skewed Student ARCH model as an alternative to both the skewed Student APARCH model and the RiskMetrics model. The skewed Student ARCH model has never been presented before and this new model combines features from the ARCH(p) model and the skewed Student density distribution. An important advantage of the skewed Student ARCH model is that its estimation does not require non-linear optimization procedures and could be routinely programmed in a ‘simple’ spreadsheet environment such as Excel. As such it is a good alternative to the RiskMetrics model (albeit more difficult to use) and it takes into account the fat-tails and skewness features of the returns.

The rest of the paper is organized in the following way. In Section 2, we briefly review the notion of risk management in commodity markets. Section 3 describes the data while Section 4 presents the VaR models that are used in the empirical analysis. The empirical application for the six commodities is given in Section 5. Section 6 concludes and presents possible new research directions.

2 Risk management in commodity markets

Fluctuations of prices in commodity markets are mainly caused by supply and demand imbalances which originate from the business cycle (energy products, metals, agricultural commodities), political events (energy products) or unexpected weather patterns (agricultural commodities). For example, the price of crude oil briefly spiked to more than \$35 a barrel in response to the Iraqi

the trader hopes that the price will fall, so that he can then buy the asset at a lower price and give it back to the lender.

invasion of Koweit at the end of 1990. A couple of months later and with the defeat of Iraq, the price of crude oil was back to less than \$20. Commodity markets are also characterized by the widespread use of futures and forward contracts, i.e. future delivery of the commodity for a price agreed up today. Indeed, derivative contracts such as futures and options were first introduced in agricultural markets as a way to hedge risk for the buyers and sellers of such products. Simple types of such contracts were even in use during the Middle Ages and Antiquity.³

Because unexpected price changes are fundamentally determined by supply and demand imbalances, market participants in commodity markets strongly focus on economic models which relate supply and demand to ‘fundamental market variables’. Moreover commodity markets are strongly shaped by storage limitations, convenience yield and seasonality effects. Next to the price changes which originate from fundamental supply and demand imbalances, price volatility can stem from the behavior of some market participants who engage in (short-term) speculation. Examples are the hoarding of the commodity in anticipation of a future rise in price, or the short-selling of futures contracts if a bear market is expected. Famous examples include the cornering of the silver market by the Hunt brothers in 1980.

Modelling risk for commodity products thus presents an inherent complexity due to the strong interaction between the trading of the products and the supply and demand imbalances which stem from the state of the economy. In this paper we characterize market risk for commodity products using a univariate time-series approach that focuses on the modelling of the VaR based on the available history of the commodity prices. As such our aim is not to quantify risk using a specific economic model tailored to forecast supply and demand in the given market, but we wish to put forward a general statistical model that accounts for the characteristics of the series of returns (for example fat-tails, skewness, heteroscedasticity) and adequately forecasts the market risk at a short-term time horizon. As such, our focus on a short-term time horizon is consistent with the use of a ‘pure’ statistical method where a more fundamental economic model would be of little use regarding 1-day risk forecasts. Over long-term time horizons (for example a couple of months), the added value of a pure statistical model would probably be much less compared to the information given by models which forecast supply and demand.

3 Data

In the empirical application of the paper we consider daily data for a collection of commodities spanning metal, energy and agricultural products. While full empirical and estimation results are presented in Section 5, we already introduce and briefly characterize our datasets at this stage to put forward the salient statistical properties of the series. More specifically we consider the

³See for example Bernstein (1996).

following commodities:

- metals: aluminium, copper and nickel daily cash prices for the 3/1/1989 - 31/1/2002 period;⁴
- energy commodities: Brent and WTI crude oil daily spot prices, for the 20/5/1987 - 18/3/2002 period;⁵
- agricultural commodity: cocoa futures contracts, daily prices for the nearest futures contract, 3/1/1994 - 31/1/2002 period.⁶

For all price series p_t , daily returns (expressed in %) are defined as $r_t = 100 [\ln(p_t) - \ln(p_{t-1})]$. Descriptive characteristics for the returns series are given in Table 1 while descriptive graphs (price, daily returns, density of the daily returns and QQ-plot against the normal distribution) are given in Figures 1-6. Note that, for the Brent and WTI crude oil, prices were extremely volatile around the Gulf war period, which led to a succession of extremely large positive and negative returns within a very short time span. A return of -40.64% was even observed on January 17th, 1991 for the WTI crude oil as the price per barrel fell from \$32.25 to \$20.28! The Brent crude oil spot price is slightly less volatile but tracks very closely the WTI crude oil price. Volatility clustering is immediately apparent from the graphs of daily returns which suggests the presence of heteroscedasticity. The density graphs and the QQ-plot against the normal distribution show that all return distributions exhibit fat tails. Moreover, the QQ-plots indicate that fat tails are not symmetric. Oil prices show the greatest volatility and excess kurtosis, and the corresponding returns are negatively skewed. Returns for the cocoa contracts are positively skewed but their excess kurtosis is rather small. Aluminium and copper cash prices are the least volatile series. This short but important preliminary descriptive and graphical analysis of the series indicate that the chosen statistical model should take into account the volatility clustering, fat tails and skewness features of the returns. In the rest of the paper and as motivated in Section 4, we focus on the skewed Student density distribution for which recent research work shows that it adequately accounts for these salient features of financial and commodity markets.⁷

4 ARCH-type VaR models

Our empirical analysis in Section 3 has shown that the returns series for the commodities we deal with exhibit heteroscedasticity. Therefore it seems natural to focus on ARCH type models to characterize the conditional variance. In this section we present parametric VaR models of the

⁴Source: London Metal Exchange.

⁵Source: U.S. Department of Energy, Energy information Administration.

⁶Source: New York Board of Trade.

⁷See Mittnik and Paoletta (2000) or Giot and Laurent (2002).

Table 1: Descriptive statistics

	Aluminium	Copper	Nickel	Brent	WTI	Cocoa
Annual s.d.	20.56	25.15	30.60	38.26	40.18	29.60
Skewness	-0.029	0.010	-0.059	-1.016	-1.298	0.438
Excess Kurtosis	4.32	6.32	3.69	19.81	23.02	2.32
Minimum	-9.78	-10.30	-11.58	-36.12	-40.64	-9.03
Maximum	7.53	15.67	10.66	17.33	18.87	9.96
$Q^2(10)$	614.4	703.5	587.5	391.7	186.9	117.4

Descriptive statistics for the daily returns (expressed in %) of the corresponding commodity. For the aluminium, copper, nickel, Brent crude oil and WTI crude oil, we consider daily cash prices, while we deal with the prices of the nearest futures contracts for the cocoa. $Q^2(10)$ is the Ljung-Box Q-statistic of order 10 on the squared series. Annual s.d. is the annualized standard deviation computed from the daily returns. All values are computed using PcGive.

ARCH class. ARCH class models were first introduced by Engle (1982) with the ARCH model. Since then, numerous extensions have been put forward⁸, but they all model the conditional variance as a function of past (squared) returns. Because quantiles are direct functions of the variance in parametric models, ARCH class models immediately translate into VaR models. As indicated in Christoffersen and Diebold (2000), volatility forecastability (such as featured by ARCH class models) decays quickly with the time horizon of the forecasts. An immediate consequence is that volatility forecastability is relevant for short time horizons (such as daily trading), but not for long time horizons on which portfolio managers usually focus. Thus, conditional VaR models of the ARCH-type should fare well when it comes to characterizing short term risk.

A complete review of all ARCH-type VaR models is outside the scope of this paper. That's why we focus on a sub-set of the possible models. More precisely, we characterize two symmetric (RiskMetrics and Student APARCH) models and two asymmetric (skewed Student APARCH and skewed Student ARCH) volatility models. We deal quite extensively with the APARCH-type model as it encompasses most of the GARCH-type models. While the RiskMetrics model is known to be a rather poor candidate for adequate forecasting of the quantiles in the presence of fat tail distribution of returns, it is nevertheless widely used by practitioners. We stress that, by symmetric and asymmetric models, we mean a possible asymmetry in the distribution of the error term (i.e. whether it is skewed or not), and not the asymmetry in the relationship between the conditional variance and the lagged squared innovations (the APARCH model features this kind of 'conditional' asymmetry whatever the chosen error term).

⁸See Engle (1995), Bera and Higgins (1993) or Palm (1996), among others, for a thorough review of the possible models.

To characterize the models, we consider a collection of daily returns, r_t , with $t = 1 \dots T$. Because daily returns are known to exhibit some serial autocorrelation⁹, we fit an AR(p) structure on the r_t series for all specifications:

$$r_t = \rho_0 + \rho_1 r_{t-1} + \dots + \rho_p r_{t-p} + e_t. \quad (1)$$

We now consider several specifications for the conditional variance of e_t .

4.1 Four volatility models

4.1.1 RiskMetrics

The RiskMetrics model is equivalent to an IGARCH model (with the normal distribution) where the autoregressive parameter is set at a pre-specified value $\lambda = 0.94$ and the coefficient of e_{t-1}^2 is equal to $1 - \lambda$:

$$e_t = \epsilon_t h_t \quad (2)$$

where ϵ_t is IID $N(0, 1)$ and h_t^2 is defined as:

$$h_t^2 = (1 - \lambda)e_{t-1}^2 + \lambda h_{t-1}^2. \quad (3)$$

As indicated in the introduction, the long side of the daily VaR is defined as the VaR level for traders having long positions in the relevant commodity: this is the ‘usual’ VaR where traders incur losses when negative returns are observed. Correspondingly, the short side of the daily VaR is the VaR level for traders having short positions, i.e. traders who incur losses when the price of the commodity increases. How good a model is at predicting long VaR is thus related to its ability to model large negative returns, while its performance regarding the short side of the VaR is based on its ability to model large positive returns.

For the RiskMetrics model, the one-step-ahead VaR as computed in $t - 1$ for long trading positions is given by $z_\alpha h_t$, for short trading positions it is equal to $z_{1-\alpha} h_t$, with z_α being the left quantile at $\alpha\%$ for the normal distribution and $z_{1-\alpha}$ is the right quantile at $\alpha\%$.¹⁰

4.1.2 Student APARCH

The APARCH(1,1) model (Ding, Granger, and Engle, 1993) is an extension of the GARCH(1,1) model of Bollerslev (1986) and specifies the conditional variance as:

⁹The serial autocorrelation found in daily returns is not necessarily at odds with the efficient market hypothesis. See Campbell, Lo, and MacKinlay (1997) for a detailed discussion.

¹⁰All VaR expressions are reported for the residuals e_t , which is equivalent to reporting the VaR centered around the expected return based on (1).

$$h_t^\delta = \omega + \alpha_1 (|e_{t-1}| - \alpha_n e_{t-1})^\delta + \beta_1 h_{t-1}^\delta, \quad (4)$$

where $\omega, \alpha_1, \alpha_n, \beta_1$ and δ are parameters to be estimated. δ ($\delta > 0$) plays the role of a Box-Cox transformation of h_t , while α_n ($-1 < \alpha_n < 1$), reflects the so-called leverage effect. A positive (respectively negative) value of α_n means that past negative (respectively positive) shocks have a deeper impact on current conditional volatility than past positive shocks (see Black, 1976; French, Schwert, and Stambaugh, 1987; Pagan and Schwert, 1990). Because the distribution of asset returns usually feature ‘fat tails’ (see Bauwens and Giot, 2001 or Alexander, 2001), we work with the Student APARCH (or t APARCH) where $e_t = \epsilon_t h_t$ and ϵ_t is IID $t(0, 1, \nu)$; h_t is defined as in (4).

For the Student APARCH model, the one-step-ahead VaR as computed in $t-1$ for long trading positions is given by $t_{\alpha, \nu} h_t$, for short trading positions it is equal to $t_{1-\alpha, \nu} h_t$, with $t_{\alpha, \nu}$ being the left quantile at $\alpha\%$ for the Student distribution with ν degrees of freedom and $t_{1-\alpha, \nu}$ is the right quantile at $\alpha\%$.

4.1.3 Skewed Student APARCH

According to Lambert and Laurent (2001) who express the skewed Student density in terms of the mean and the variance¹¹, the innovation process ε is said to be $SKST(0, 1, \xi, \nu)$, i.e. (standardized) skewed Student distributed, if:

$$f(\epsilon|\xi, \nu) = \begin{cases} \frac{2}{\xi + \frac{1}{\xi}} sg[\xi(s\epsilon + m)|\nu] & \text{if } \epsilon < -\frac{m}{s} \\ \frac{2}{\xi + \frac{1}{\xi}} sg[(s\epsilon + m)/\xi|\nu] & \text{if } \epsilon \geq -\frac{m}{s} \end{cases}, \quad (5)$$

where $g(\cdot|\nu)$ is the symmetric (unit variance) Student density and ξ is the asymmetry coefficient;¹² m and s^2 are respectively the mean and the variance of the non-standardized skewed Student and are defined as follows:

$$m = \frac{\Gamma\left(\frac{\nu-1}{2}\right) \sqrt{\nu-2}}{\sqrt{\pi} \Gamma\left(\frac{\nu}{2}\right)} \left(\xi - \frac{1}{\xi}\right) \quad (6)$$

and

$$s^2 = \left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2. \quad (7)$$

Note also that for given values of ξ and ν and a probability α , the quantile function is:

¹¹See also Fernández and Steel (1998).

¹²The asymmetry coefficient $\xi > 0$ is defined such that the ratio of probability masses above and below the mean is $\frac{\Pr(\epsilon \geq 0|\xi)}{\Pr(\epsilon < 0|\xi)} = \xi^2$.

$$st_{\alpha,v,\xi} = \begin{cases} \frac{\frac{1}{\xi}t_1 - m}{s} & \text{if } \alpha < \frac{1}{1+\xi^2} \\ \frac{-\xi t_2 - m}{s} & \text{if } \alpha \geq \frac{1}{1+\xi^2} \end{cases} \quad (8)$$

where $t_1 = t_{\frac{\alpha}{2}(1+\xi^2),v}$ and $t_2 = t_{\frac{1-\alpha}{2}(1+\xi^2),v}$

For the skewed Student APARCH model (whose conditional variance is modelled as in Equation (4)), the VaR for long and short positions is given by $st_{\alpha,v,\xi} h_t$ and $st_{1-\alpha,v,\xi} h_t$, with $st_{\alpha,v,\xi}$ being the left quantile at $\alpha\%$ for the skewed Student distribution with v degrees of freedom and asymmetry coefficient ξ ; $st_{1-\alpha,v,\xi}$ is the corresponding right quantile. If $\log(\xi)$ is smaller than zero (or $\xi < 1$), $|st_{\alpha,v,\xi}| > |st_{1-\alpha,v,\xi}|$ and the VaR for long traders will be larger (for the same conditional variance) than the VaR for short traders. When $\log(\xi)$ is positive, we have the opposite result.

4.1.4 Skewed Student ARCH

The estimation of the APARCH skewed Student is done by (approximate) maximum likelihood and thus requires the use of non-linear optimization techniques. While non-linear optimization methods are available in advanced statistical and econometric packages, these estimation techniques are not readily available to market participants involved in the trading and risk management of the financial or commodity assets. This has led to a widespread use of the RiskMetrics approach in VaR applications as this simple but easy-to-use method does not require advanced estimation procedures. However, it is widely acknowledged that RiskMetrics is not good at modelling the non-normal density distribution of returns and only models approximatively the observed volatility clustering.

In this sub-section, we introduce a statistical model that takes into account volatility clustering, fat tails and skewness features but does not require non-linear estimation methods. In his seminal paper, Engle (1982) suggests to estimate the ARCH(q) model in a somewhat simpler four-step procedure based on the method of scoring. Let us briefly review this estimation procedure. Equation (1) can be rewritten in matrix notation as:

$$r_t = R_t \Gamma + e_t, \quad (9)$$

where $R_t = (1, r_{t-1}, \dots, r_{t-q})$ and $\Gamma = (\rho_0, \rho_1, \dots, \rho_p)'$. The ARCH(q) model is defined as:

$$h_t^2 = \omega + \alpha_1 e_{t-1}^2 + \dots + \alpha_q e_{t-q}^2, \quad (10)$$

or in matrix notation:

$$h_t^2 = E_t \Delta, \quad (11)$$

where $E_t = (1, e_{t-1}^2, \dots, e_{t-q}^2)$ and $\Delta = (\omega, \alpha_1, \dots, \alpha_q)'$.

Estimation of the model described by Equations (9)-(11) can be achieved in four steps:

- First, compute $\tilde{\Gamma} = (R'R)^{-1}R'r$ and $\tilde{e} = r - R\tilde{\Gamma}$ using the T observations. $\tilde{\Gamma}$ is a consistent, but inefficient, estimator of Γ .
- Second, using observations $1 + q$ to T , regress \tilde{e}_t^2 on a constant and $\tilde{e}_{t-1}^2, \dots, \tilde{e}_{t-q}^2$ to obtain $\tilde{\Delta}$.
- Third, using $\tilde{\Delta}$, compute: *i*) $\tilde{h}_t^2 = \tilde{E}_t\tilde{\Delta}$, where $\tilde{E}_t = (1, \tilde{e}_{t-1}^2, \dots, \tilde{e}_{t-q}^2)$; *ii*) $g_t = \frac{\tilde{e}_t^2}{\tilde{h}_t^2} - 1$; *iii*) $z_{1,t} = \frac{1}{\tilde{h}_t^2}$ and for observations $1 + q$ to T , *iv*) $z_{2,t} = \left(\frac{\tilde{e}_{t-1}^2}{\tilde{h}_t^2}, \dots, \frac{\tilde{e}_{t-q}^2}{\tilde{h}_t^2}\right)$. Then, collect $T - q$ observations in $g = [g_t]_{t=q+1, \dots, T}$ and $Z = [z_{1,t}, z_{2,t}]_{t=q+1, \dots, T}$ and compute $\bar{\Delta} = (Z'Z)^{-1}Z'g$. The asymptotically efficient estimator of Δ is $\hat{\Delta} = \tilde{\Delta} + \bar{\Delta}$. Its asymptotic covariance matrix is estimated as $2(Z'Z)^{-1}$.
- Finally, compute *i*) $\hat{h}_t^2 = \tilde{E}_t\hat{\Delta}$ for observations $1 + q$ to T . Then for observations $1 + q$ to $T - q$, compute *ii*) $f_t = \hat{h}_t^{-2} + 2\tilde{e}_t^2 \sum_{j=1}^q \hat{\alpha}_j \hat{h}_{t+j}^{-4}$ and *iii*) $s_t = \hat{h}_t^{-2} - \sum_{j=1}^q \hat{\alpha}_j \hat{h}_{t+j}^{-2} \left(\frac{\tilde{e}_{t+j}^2}{\hat{h}_{t+j}^2} - 1\right)$. An asymptotically efficient estimator of Γ is given by $\hat{\Gamma} = \tilde{\Gamma} + \bar{\Gamma}$, where $\bar{\Gamma} = (W'W)^{-1}W'v$, $v = [e_t s_t / f_t]_{t=q+1, \dots, T-q}$ and $W = [f_t X_t]_{t=1, \dots, T-q}$. Its asymptotic covariance matrix is estimated as $(W'W)^{-1}$.

This estimator is asymptotically equivalent to maximum likelihood under the Gaussian assumption and thus one could compute, as in Section 4.1.1, the VaR forecast using the corresponding Gaussian quantile. If this assumption does not hold (which is more than likely given the observed statistical properties of the returns series), this estimator is still consistent and can be considered as a quasi-maximum likelihood estimator.¹³ However, the consistency of the conditional mean and conditional variance does not mean that the model will provide accurate VaR forecasts. Indeed, for daily stock returns, Giot and Laurent (2002) and Mittnik and Paoletta (2000) show that a normal APARCH model delivers poor (in-sample and out-of-sample) VaR forecasts and that accounting for the observed skewness and kurtosis is crucial.

Therefore, we propose to use an adaptive method to compute the one-day-ahead VaR. Let us first define the estimated standardized residuals as: $\hat{\epsilon}_t = (r_t - R_t \hat{\Gamma}) / \hat{h}_t$ and $SK(\hat{\epsilon}_t)$ and $KU(\hat{\epsilon}_t)$ the empirical skewness and kurtosis coefficients of $\hat{\epsilon}_t$. If $\hat{\epsilon}_t$ is normally distributed, $SK(\hat{\epsilon}_t)$ and $KU(\hat{\epsilon}_t)$ should not be significantly different from zero and three, respectively.¹⁴ Accordingly and following Lambert and Laurent (2001), if $\hat{\epsilon}_t \sim SKST(0, 1, \xi, \nu)$:

$$SK(\hat{\epsilon}_t | \xi, \nu) = \frac{E(\hat{\epsilon}_t^3 | \xi, \nu) - 3E(\hat{\epsilon}_t | \xi, \nu)E(\hat{\epsilon}_t^2 | \xi, \nu) + 2E(\hat{\epsilon}_t | \xi, \nu)^3}{Var(\hat{\epsilon}_t | \xi, \nu)^{\frac{3}{2}}} \quad (12)$$

¹³In this case, the asymptotic covariance matrix has to be corrected accordingly. We will not give the technical details here since we are only concerned with VaR forecasts.

¹⁴Lambert and Laurent (2001) and Giot and Laurent (2002) have shown that, for various financial daily returns, it is realistic to assume that $\hat{\epsilon}_t$ is skewed Student distributed (see previous section).

$$KU(\hat{\epsilon}_t|\xi) = \frac{E(\hat{\epsilon}_t^4|\xi, v) - 4E(\hat{\epsilon}_t|\xi, v)E(\hat{\epsilon}_t^3|\xi, v) + 6E(\hat{\epsilon}_t^2|\xi, v)E(\hat{\epsilon}_t|\xi, v)^2 - 3E(\hat{\epsilon}_t|\xi, v)^4}{Var(\hat{\epsilon}_t|\xi, v)^2} \quad (13)$$

where $E(\hat{\epsilon}_t^r|\xi, v) = M_{r,v} \frac{\xi^{r+1} + \frac{(-1)^r}{\xi^{r+1}}}{\xi + \frac{1}{\xi}}$, $M_{r,v} = \int_0^\infty 2s^r g(s|v) ds$, and $M_{r,v}$ is the r^{th} order moment of $g(\cdot|v)$ truncated to the positive real values.¹⁵ In other words, for given values of ξ and v , the skewness and kurtosis are given by Equations (12) and (13) respectively. Note that when $\xi = 1$ and $v = +\infty$ we get the skewness and the kurtosis of the Gaussian density and when $\xi = 1$ but $v > 2$, we have the skewness and the kurtosis of the (standardized) Student distribution.

Our goal in this paper is thus to put forward a tractable solution for estimating ξ and v (i.e. an estimation method that does not require non-linear optimization techniques) and use these values to compute the corresponding quantile at $\alpha\%$. This cannot be achieved by solving the system constituted by Equations (12) and (13) with respect to ξ and v since these equations are highly non-linear in ξ and v . The solution we suggest is the following simple grid search method:

- a) generate skewness and kurtosis combinations, i.e. $Sk(\xi_i, v_i)$ and $Ku(\xi_i, v_i)$ by letting ξ vary between respectively $\xi_{min} (> 0)$ and $\xi_{max} (\in \mathfrak{R}^+)$ and v between $v_{min} (> 4)$ and $v_{max} (\in \mathfrak{R}^+)$, by choosing sufficiently small increments in the sequences of ξ_i and v_i values.¹⁶
- b) define the optimal ξ and v parameters as the parameters that minimize $|Sk(\hat{\epsilon}_t) - Sk(\xi_i, v_i)| + |Ku(\hat{\epsilon}_t) - Ku(\xi_i, v_i)|$.

From a computational point of view, our procedure thus boils down to a regression approach (to determine h_t and model heteroscedasticity) combined with a search-on-grid method to compute the optimal ξ and v parameters so that our model takes into account the observed skewness and kurtosis of the data. Both steps can easily be implemented in a simple statistical package or spreadsheet environment and they do not require non-linear optimization modules (such as the ML or CML packages in GAUSS for example, which is needed for the estimation of the skewed Student APARCH model).

With respect to the simple RiskMetrics method, we thus substitute a regression approach (to determine h_t) to the $h_t^2 = (1 - \lambda)e_{t-1}^2 + \lambda h_{t-1}^2$ fixed rule and we add the second step to determine the parameters of the skewed Student density distribution (to switch from the normal distribution as in RiskMetrics to a more general density distribution). Finally, the VaR can be computed as in the previous sub-section, i.e. the VaR for long and short positions is given by $st_{\alpha, v, \xi} h_t$

¹⁵As indicated above, $g(\cdot|v)$ is the (standardized) Student probability density function, with number of degrees of freedom $v > 2$.

¹⁶In the empirical application, we set $\xi_{min} = 1$ and $\xi_{max} = 2$ with an increment of 0.01 and $v_{min} = 4.1$ and $v_{max} = 25$ with an increment of 0.05. This generates 42,521 combinations of skewness and kurtosis. Note that we only generate positive (or zero) skewness ($\xi \geq 1$) since negative skewness can be recovered by taking $1/\xi$.

$st_{1-\alpha,v,\xi} h_t$, with $st_{\alpha,v,\xi}$ being the left quantile at $\alpha\%$ for the skewed Student distribution with v degrees of freedom and asymmetry coefficient ξ ; $st_{1-\alpha,v,\xi}$ is the corresponding right quantile. Parameters ξ and v are given by the outcome of step b) in the minimization procedure described above.

5 Empirical application

5.1 Out-of-sample VaR forecasts

In an actual risk management setting, VaR models deliver out-of-sample forecasts, where the model is estimated on the known returns (up to time t for example) and the VaR forecast is made for period $[t + 1, t + h]$, where h is the time horizon of the forecasts. In this subsection we implement this testing procedure for the long and short VaR with $h = 1$ day.¹⁷ We use an iterative procedure where the RiskMetrics, skewed Student APARCH and skewed Student ARCH models are estimated to predict the one-day-ahead VaR. For all models, an AR(3) structure is fitted on the returns and an APARCH(1,1) structure successfully takes into account the heteroscedasticity. Regarding the skewed Student ARCH model, a lag structure of around 10 lags is needed to model the heteroscedasticity.

In our iterative procedure, the first estimation sample is the complete sample for which the data is available less the last five years. The predicted one-day-ahead VaR (both for long and short positions) is then compared with the observed return and both results are recorded for later assessment using the statistical tests. At the second iteration, the estimation sample is augmented to include one more day, the models are *re-estimated* and the VaRs are forecasted and recorded.¹⁸ We iterate the procedure until all days (less the last one) have been included in the estimation sample.¹⁹ Corresponding failure rates and VaR violations are then computed by comparing the long and short forecasted VaR_{t+1} with the observed return e_{t+1} for all days in the five-year period.²⁰ In our application, we define a failure rate f_l for the long traders, which is equal to the percentage of negative returns smaller than one-step-ahead VaR for long positions. Correspondingly, we define f_s as the failure rate for short traders as the percentage of positive returns larger than the one-step-ahead VaR for short positions.

¹⁷Prior to the out-of-sample tests, we also implemented in-sample VaR forecasts. We do not include those results in the paper as they are similar to the ones reported for the out-of-sample forecasts.

¹⁸Note that, at this stage, we do not assume any stability hypothesis on the coefficients as the models are re-estimated each time a new observation enters the information set.

¹⁹Note that an alternative strategy would be to use rolling-sample estimation, where the estimation sample is of fixed length and where older returns are no longer taken into account as new observations are included.

²⁰By definition, the failure rate is the number of times returns exceed (in absolute value) the forecasted VaR. If the VaR model is correctly specified, the failure rate should be equal to the pre-specified VaR level.

Table 2: Out-of-sample VaR results

	VaR for long positions					VaR for short positions				
	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%
Aluminium										
RiskMetrics	0.897	0.928	0.092	0.003	0.002	0.103	0.006	0.001	0.001	0.001
st APARCH	1	0.785	0.910	0.904	0.487	0.609	0.928	0.083	0.590	0.487
st ARCH	0.513	0.928	0.864	0.904	0.487	0.897	0.306	0.163	0.325	0.487
Copper										
RiskMetrics	0.513	0.427	0.054	0.089	0.153	0.609	0.071	0	0	0.002
st APARCH	0.291	0.785	0.643	0.784	0.932	0.431	0.785	0.283	0.590	0.156
st ARCH	0.006	0.158	0.864	0.784	0.487	0.291	0.656	0.445	0.590	0.487
Nickel										
RiskMetrics	0.696	0.656	0.509	0.019	0.007	0.523	0.009	0	0	0
st APARCH	1	0.306	0.445	0.783	0.645	0.166	0.101	0.237	0.515	0.932
st ARCH	0.374	0.535	0.643	0.784	0.645	0.003	0	0.016	0.090	0.337
Cocoa										
RiskMetrics	0.795	0.535	0.509	0.515	0.153	0.445	0.006	0	0	0
st APARCH	0.207	0.256	0.910	0.784	0.337	0.207	0.192	0.054	0.311	0.645
st ARCH	0.001	0.009	0.151	0.090	0.007	0.255	0.101	0.030	0.515	0.337
Brent crude oil										
RiskMetrics	0.523	0.656	0.054	0.043	0.001	0.609	0.535	0.054	0.008	0.001
st APARCH	0.310	0.785	0.697	0.904	0.487	0.255	0.656	0.910	0.311	0.932
st ARCH	0	0.003	0.237	0.311	0.337	0.007	0.014	0.030	0.019	0.645
WTI crude oil										
RiskMetrics	0.374	0.003	0.004	0	0	1	0.656	0.091	0.003	0.001
st APARCH	0.166	0.656	0.054	0.019	0.645	0.166	0.535	0.697	0.783	0.153
st ARCH	0.061	0.192	0.237	0.311	0.022	0.035	0.033	0.016	0.311	0.153

P-values for the null hypothesis $f_l = \alpha$ (i.e. failure rate for the long trading positions is equal to α , top of the table) and $f_s = \alpha$ (i.e. failure rate for the short trading positions is equal to α , bottom of the table). α is equal successively to 5%, 2.5%, 1%, 0.5% and 0.25%. The models are successively the RiskMetrics, skewed Student APARCH and skewed Student ARCH models. These are out-of-sample VaR tests on the last five years of our datasets. Note that a P-value smaller than 0.05 indicates that the corresponding VaR model does not perform adequately out-of-sample.

Because the computation of the empirical failure rate defines a sequence of above VaR level/below VaR level observations (akin to draws from the binomial distribution), it is possible to test $H_0 : f = \alpha$ against $H_1 : f \neq \alpha$, where f is the failure rate (estimated by \hat{f} , the empirical failure rate).²¹ At the 5% level and if T yes/no observations are available, a confidence interval for \hat{f} is given by $[\hat{f} - 1.96\sqrt{\hat{f}(1-\hat{f})/T}, \hat{f} + 1.96\sqrt{\hat{f}(1-\hat{f})/T}]$. In this paper these tests are successively applied to the failure rate f_l for long traders and then to f_s , the failure rate for short traders. Empirical results (i.e. P-values for the Kupiec LR test) for the aluminium, copper, nickel, Brent crude oil and WTI crude oil cash prices and the nearby futures contracts for the cocoa are given in Table 2. These results indicate that:

- a) as expected, the RiskMetrics method performs rather poorly when α is at or below 1% (actually, RiskMetrics VaR results are only globally acceptable at the 5% level). Its total success rate, i.e. across the six commodities and the five theoretical α , is equal to 31/60 or 52.7%.
- b) the skewed Student APARCH model delivers excellent results as the empirical failure rates are statistically equal to their theoretical values in all cases but one. Moreover, this model performs equally well for the six commodities under study in this paper and for the long and short trading positions. Its total success rate is equal to 59/60 or 98.3%.
- c) the skewed Student ARCH model performs rather well, although its performance is sometimes disappointing for 'large' α levels because it is overly cautious and suggests larger-than-needed VaR levels (hence the empirical failure rate is smaller than the theoretical level and the null hypothesis of equality is rejected). Its total success rate is equal to 38/60 or 63.3%.
- d) the discussion in points b) and c) above shows that the APARCH structure is much more flexible than the ARCH structure. This is expected as the APARCH model allows for conditional asymmetry in the variance equation and a δ coefficient which is not constrained to a value of 2 as in the GARCH type models. Actually, this explains the rather dismal performance of the skewed Student ARCH model for the Brent and WTI crude oil prices. For these two commodities, the estimation of the skewed Student APARCH model indicates that coefficient δ is not statistically different from 1, while it is between 1 and 2 for the other commodities. Hence an APARCH skewed Student model should be used whenever non-linear optimization techniques are available as it is not clear (ex-ante) that it is the conditional variance (i.e. $\delta = 2$) or conditional standard deviation (i.e. $\delta = 1$) that should be modelled.

Based on these estimation outputs and the estimation methods discussed at length in Section

²¹In the literature on VaR models, this test is also called the Kupiec LR test, if the hypothesis is tested using a likelihood ratio test. See Kupiec (1995).

Table 3: Advantages and disadvantages of the VaR estimation methods

	RiskMetrics	Skewed Student APARCH	Skewed Student ARCH
Estimation method	Fixed coefficients	Non-linear optimization Maximum likelihood	Two-step: 1) regression analysis 2) search-on-grid
Advantages	Simple	Very flexible	Flexible, ML not needed
Disadvantages	Too crude	Requires ML	Inferior to APARCH
Skewness?	No	Yes	Yes
Excess kurtosis?	No	Yes	Yes
Heteroscedasticity?	Approximately	Yes	Yes

Advantages and disadvantages of the VaR estimation methods considered in this paper.

4.1, we briefly summarize the advantages and disadvantages of the RiskMetrics, skewed Student APARCH and skewed Student ARCH models in Table 3.

Finally, we perform the same out-of-sample VaR analysis but we choose to estimate the ‘best’ VaR model (i.e. the skewed Student APARCH model) on a 10- and 25-day basis. Thus, we proceed as previously by incrementing our dataset to include the out-of-sample observations and compute the empirical failure rate, but we no longer estimate the skewed Student APARCH model daily. We estimate this model whenever a multiple of 10 (and 25) observations have been added. This corresponds to the situation where a daily estimation of the VaR model is too cumbersome (for example market practitioners who deal with a large number of assets and who do not have the time to re-run the programs each day), and the model is estimated on a lower frequency. The estimation results reported in Table 4 indicate that the skewed Student APARCH model still performs admirably at these lower estimation frequencies.

6 Conclusion

In this paper, we introduced Value-at-Risk models relevant for commodity traders who have long and short trading positions in commodity markets. Our time horizon is short-term as we focused on market risk at the one-day time horizon. In an out-of-sample study spanning metal (aluminium, copper and nickel cash prices), energy (Brent crude oil and WTI crude oil cash prices) and agricultural commodities (cocoa nearby futures contracts), we assessed the performance of the RiskMetrics, skewed Student APARCH and skewed student ARCH models. While the skewed Student APARCH model performed best in all cases, the skewed Student ARCH model never-

Table 4: Skewed Student APARCH out-of-sample VaR results for several estimation frequencies

	VaR for long positions					VaR for short positions				
	5%	2.5%	1%	0.5%	0.25%	5%	2.5%	1%	0.5%	0.25%
Aluminium										
daily	1	0.785	0.910	0.904	0.487	0.609	0.928	0.083	0.590	0.487
10-day	0.897	0.785	0.910	0.325	0.487	0.700	0.785	0.083	0.590	0.156
25-day	0.897	0.785	0.910	0.590	0.487	0.797	0.928	0.083	0.590	0.487
Copper										
daily	0.291	0.785	0.643	0.784	0.932	0.431	0.785	0.283	0.590	0.156
10-day	0.291	0.785	0.643	0.904	0.932	0.431	0.785	0.163	0.590	0.156
25-day	0.234	0.785	0.643	0.784	0.932	0.431	0.785	0.283	0.590	0.156
Nickel										
daily	1	0.306	0.445	0.783	0.645	0.166	0.101	0.237	0.515	0.932
10-day	1	0.405	0.445	0.783	0.645	0.166	0.101	0.237	0.515	0.932
25-day	1	0.405	0.445	0.783	0.645	0.166	0.101	0.237	0.515	0.932

P-values for the null hypothesis $f_l = \alpha$ (i.e. failure rate for the long trading positions is equal to α , top of the table) and $f_s = \alpha$ (i.e. failure rate for the short trading positions is equal to α , bottom of the table). α is equal successively to 5%, 2.5%, 1%, 0.5% and 0.25%. The skewed Student APARCH is estimated daily and on a 10- and 25-day basis to compute the out-of-sample forecasts on the last five years of our datasets. Note that a P-value smaller than 0.05 indicates that the corresponding VaR model does not perform adequately out-of-sample.

theless delivered good results. An important advantage of the skewed Student ARCH model is that its estimation does not require numerical optimization procedures and could be routinely programmed in a ‘simple’ spreadsheet environment such as Excel.

Several extensions of our study can be considered. The most obvious one is the ability of the models to correctly forecast out-of-sample VaR at a time horizon longer than one day. As mentioned in Section 2, commodity prices over the long-run are fundamentally determined by the economic cycles and availability of resources. Nevertheless it could be interesting to assess the performance of our models in a one-week or one-month risk framework. Secondly and with a very active market for commodity derivatives, implied volatility from short-term options (or options on futures) is readily available. Thus one could look at the relevance of the additional information provided by the implied volatility and assess how it improves on the information given by the past returns in the APARCH specification for example.²² See also Giot (2003) for a recent application to agricultural commodity markets.

References

- ALEXANDER, C. (2001): *Market models*. Wiley.
- BAUWENS, L., AND P. GIOT (2001): *Econometric modelling of stock market intraday activity*. Kluwer Academic Publishers.
- BERA, A., AND M. HIGGINS (1993): “ARCH models: properties, estimation and testing,” *Journal of Economic Surveys*.
- BERNSTEIN, P. (1996): *Against the gods: the remarkable story of risk*. Wiley.
- BLACK, F. (1976): “Studies of stock market volatility changes,” *Proceedings of the American Statistical Association, Business and Economic Statistics Section*, pp. 177–181.
- BLAIR, B., S. POON, AND S. TAYLOR (2001): “Forecasting S&P100 volatility: the incremental information content of implied volatilities and high-frequency index returns,” *Journal of Econometrics*, 105, 5–26.
- BOLLERSLEV, T. (1986): “Generalized autoregressive conditional heteroskedasticity,” *Journal of Econometrics*, 31, 307–327.
- CAMPBELL, J., A. LO, AND A. MACKINLAY (1997): *The econometrics of financial markets*. Princeton University Press, Princeton.

²²Similar studies for stock index data are available in Day and Lewis (1992), Xu and Taylor (1995) or Blair, Poon, and Taylor (2001).

- CHRISTOFFERSEN, P., AND F. DIEBOLD (2000): “How relevant is volatility forecasting for financial risk management?,” *Review of Economics and Statistics*, 82, 1–11.
- DANIELSSON, J., AND C. DE VRIES (2000): “Value-at-Risk and extreme returns,” *Annales d’Economie et Statistique*, 3, 73–85.
- DAY, T., AND C. LEWIS (1992): “Stock market volatility and the information content of stock index options,” *Journal of Econometrics*, 52, 267–287.
- DING, Z., C. W. J. GRANGER, AND R. ENGLE (1993): “A long memory property of stock market returns and a new model,” *Journal of Empirical Finance*, 1, 83–106.
- DOWD, K. (1998): *Beyond Value-at-Risk*. Wiley.
- ENGLE, R. (1982): “Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation,” *Econometrica*, 50, 987–1007.
- (1995): *ARCH selected readings*. Oxford University Press, Oxford.
- FERNÁNDEZ, C., AND M. STEEL (1998): “On Bayesian modelling of fat tails and skewness,” *Journal of the American Statistical Association*, 93, 359–371.
- FRENCH, K., G. SCHWERT, AND R. STAMBAUGH (1987): “Expected stock returns and volatility,” *Journal of Financial Economics*, 19, 3–29.
- GIOT, P. (2003): “The information content of implied volatility in agricultural commodity markets,” *Forthcoming in the Journal of Futures Markets*.
- GIOT, P., AND S. LAURENT (2002): “Value-at-Risk for long and short trading positions,” *Forthcoming in the Journal of Applied Econometrics*.
- JORION, P. (2000): *Value-at-Risk*. McGraw-Hill.
- KRONER, K., K. KNEAFSEY, AND S. CLAESSENS (1994): “Forecasting volatility in commodity markets,” *Journal of Forecasting*, 14, 77–95.
- KUPIEC, P. (1995): “Techniques for verifying the accuracy of risk measurement models,” *Journal of Derivatives*, 2, 173–84.
- LAMBERT, P., AND S. LAURENT (2001): “Modelling financial time series using GARCH-type models and a skewed student density,” Mimeo, Université de Liège.
- MANFREDO, M., AND R. LEUTHOLD (1998): “Agricultural applications of Value-at-Risk analysis: a perspective,” Office for Futures and Options Research (OFOR) Paper Number 98-04.

- MITTNIK, S., AND M. PAOLELLA (2000): “Conditional density and Value-at-Risk prediction of Asian currency exchange rates,” *Journal of Forecasting*, 19, 313–333.
- PAGAN, A., AND G. SCHWERT (1990): “Alternative models for conditional stock volatility,” *Journal of Econometrics*, 45, 267–290.
- PALM, F. (1996): “GARCH models of volatility,” in *Maddala, G.S., Rao, C.R., Handbook of Statistics*, pp. 209–240.
- SAUNDERS, A. (2000): *Financial institutions management*. McGraw-Hill.
- SAUNDERS, A., AND L. ALLEN (2002): *Credit risk measurement, second edition*. Wiley.
- VAN DEN GOORBERGH, R., AND P. VLAAR (1999): “Value-at-Risk analysis of stock returns. Historical simulation, tail index estimation?,” De Nederlandse Bank-Staff Report, 40.
- VLAAR, P. (2000): “Value-at-Risk models for Dutch bond portfolios,” *Journal of Banking and Finance*, 24, 1131–1154.
- XU, X., AND S. TAYLOR (1995): “Conditional volatility and the informational efficiency of the PHLX currency options market,” *Journal of Banking and Finance*, 19, 803–821.

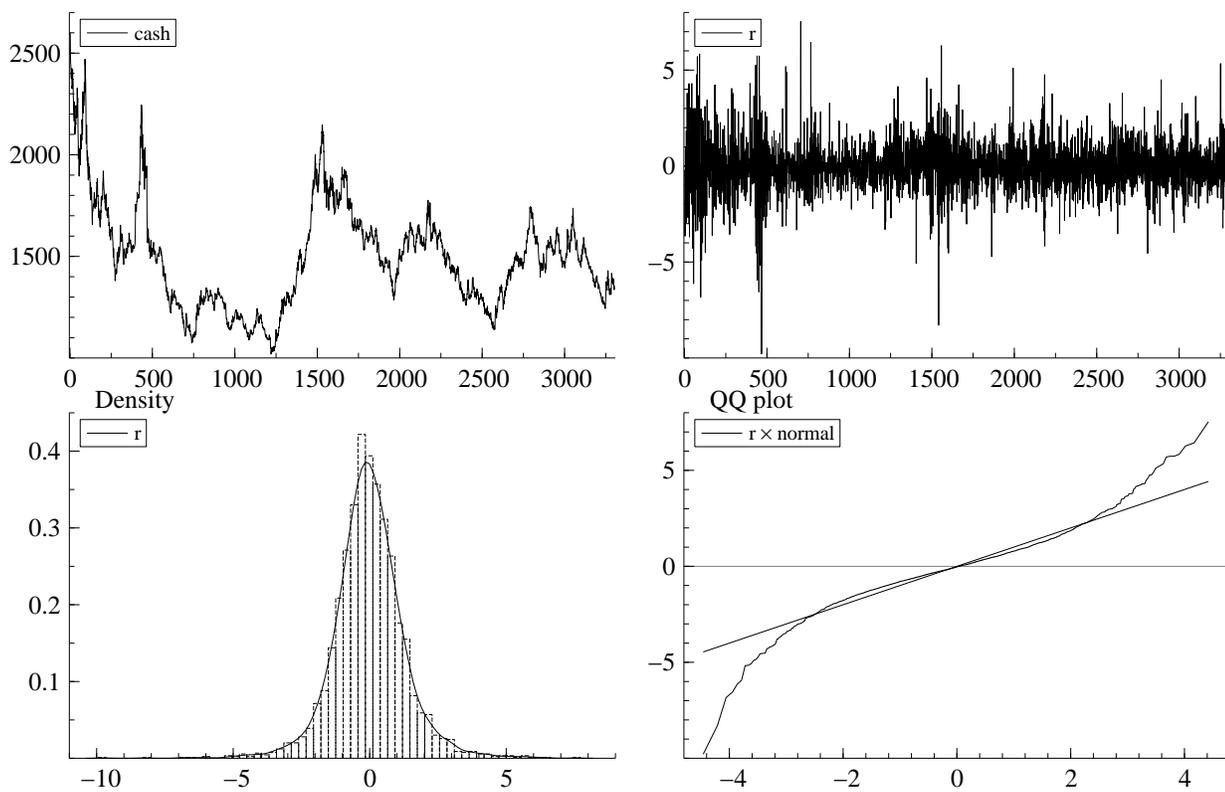


Figure 1: Aluminium cash price in level (*cash*), daily returns (*r*), daily returns density and QQ-plot against the normal distribution. The time period is 3/1/1989 - 31/1/2002.

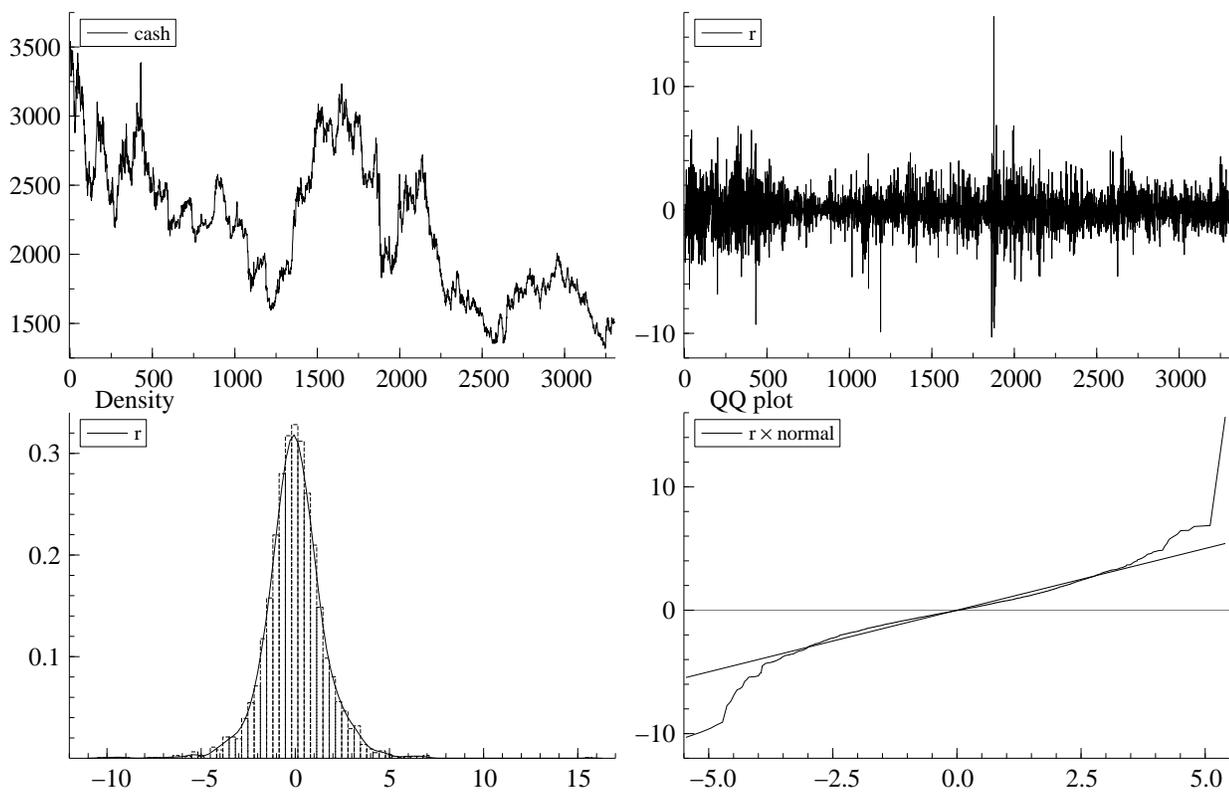


Figure 2: Copper cash price in level (*cash*), daily returns (*r*), daily returns density and QQ-plot against the normal distribution. The time period is 3/1/1989 - 31/1/2002.

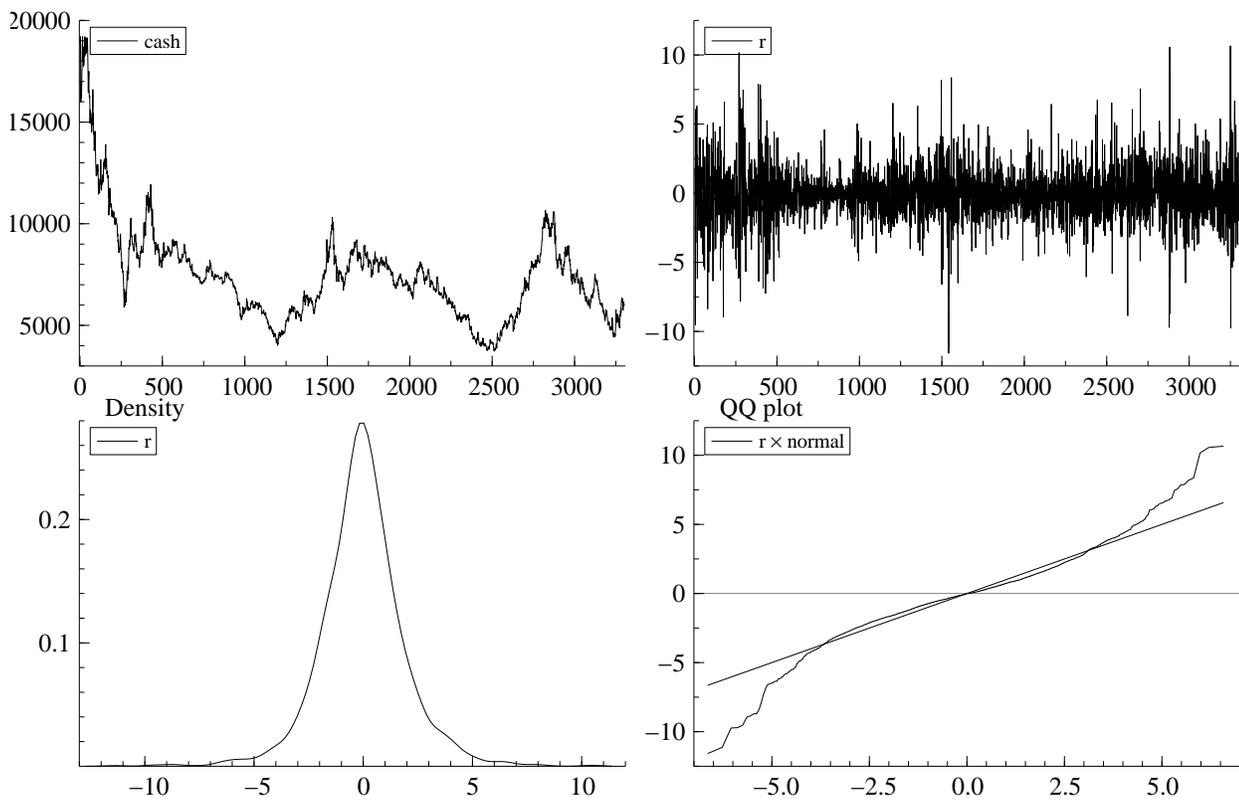


Figure 3: Nickel cash price in level (*cash*), daily returns (*r*), daily returns density and QQ-plot against the normal distribution. The time period is 3/1/1989 - 31/1/2002.

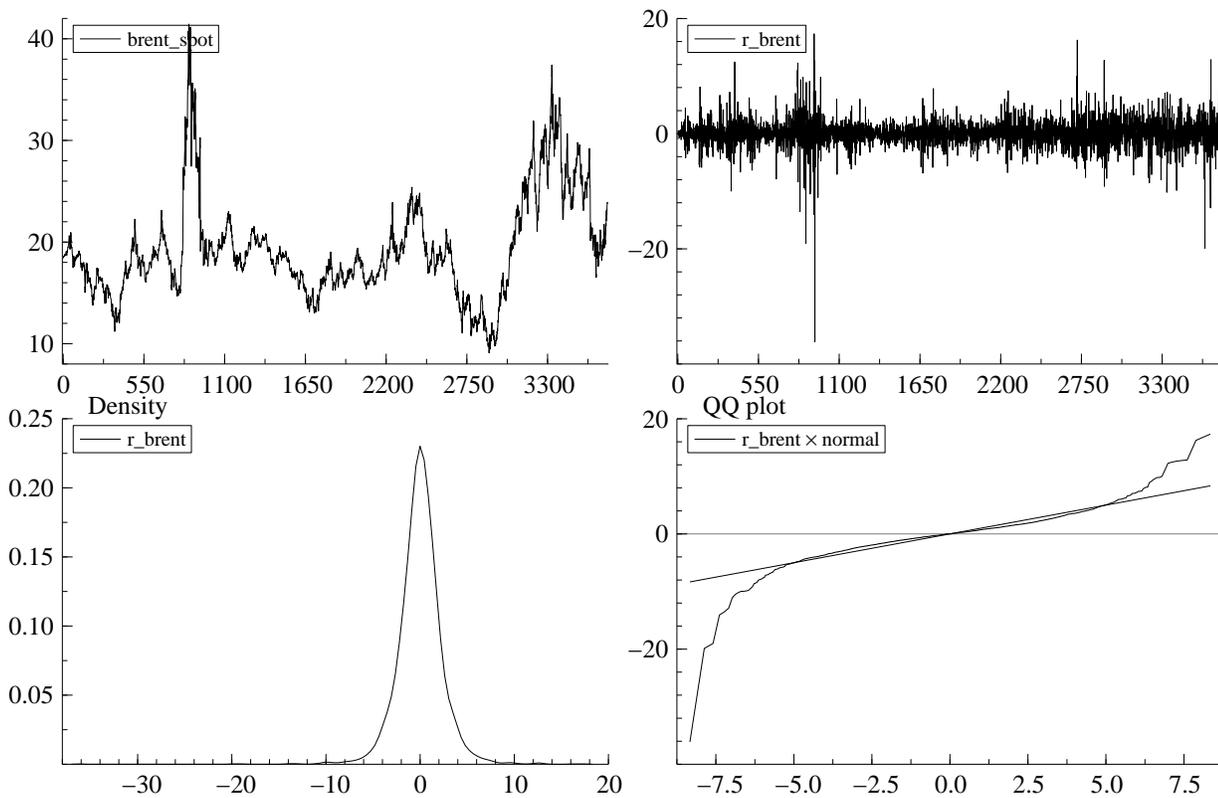


Figure 4: Brent crude oil cash price in level (*brent_spot*), daily returns (*r_brent*), daily returns density and QQ-plot against the normal distribution. The time period is 20/5/1987 - 18/3/2002.

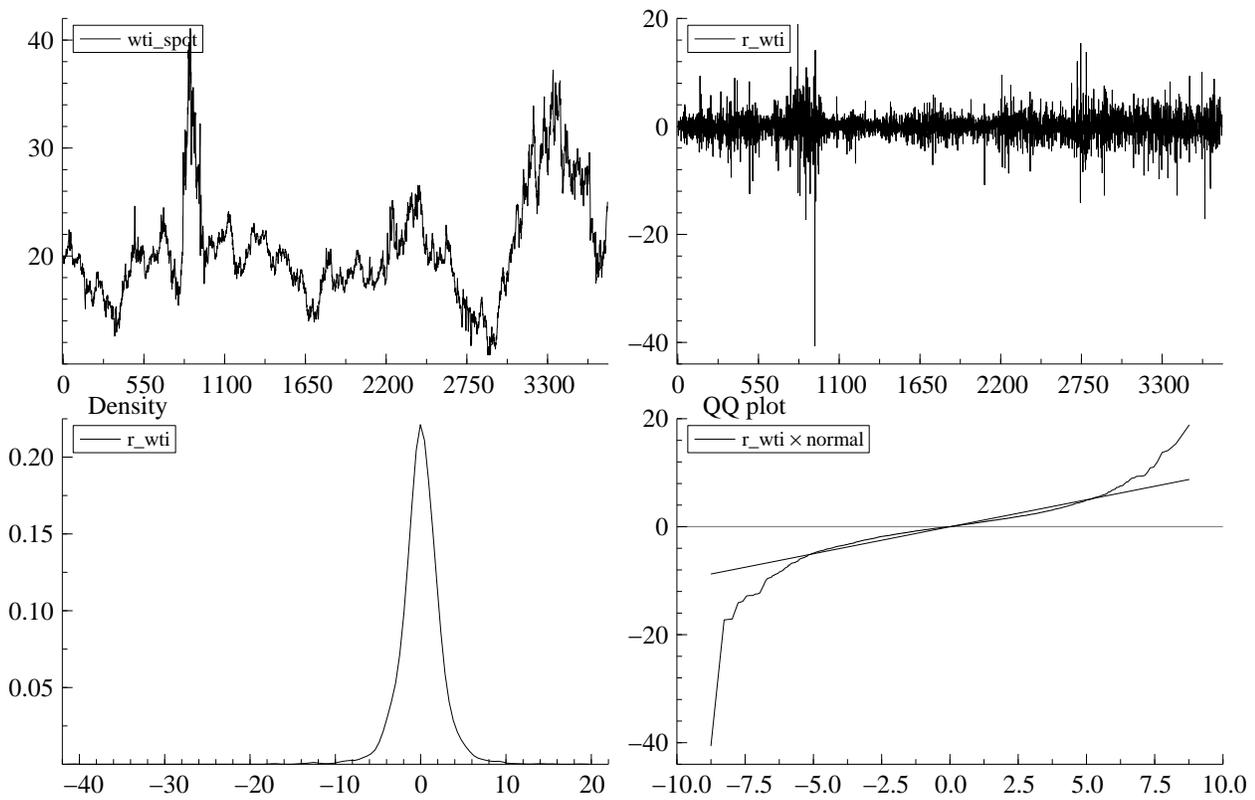


Figure 5: WTI crude oil cash price in level (wti_spot), daily returns (r_wti), daily returns density and QQ-plot against the normal distribution. The time period is 20/5/1987 - 18/3/2002.

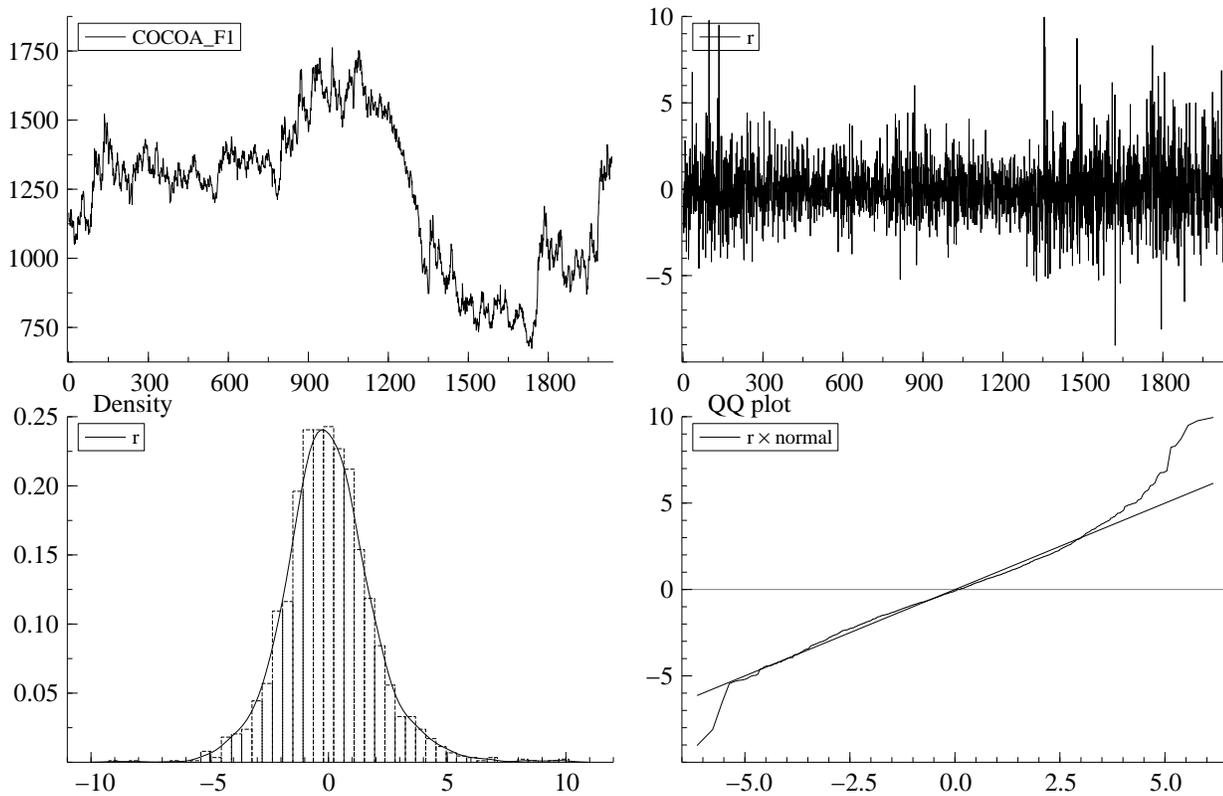


Figure 6: Nearby future cocoa price in level (*COCOA_F1*), daily returns (r), daily returns density and QQ-plot against the normal distribution. The time period is 3/1/1994 - 31/1/2002.