Robust estimation of intraweek periodicity in volatility and jump detection

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Abstract

Opening, lunch and closing of financial markets induce a periodic component in the volatility of high-frequency returns. We show that price jumps cause a large bias in the classical periodicity estimators and propose robust alternatives. We find that accounting for periodicity greatly improves the accuracy of intraday jump detection methods. It increases the power to detect the relatively small jumps occurring at times for which volatility is periodically low and reduces the number of spurious jump detections at times of periodically high volatility. We use the series of detected jumps to estimate robustly the long memory parameter of the squared EUR/USD, GBP/USD and YEN/USD returns.

Keywords: high-frequency foreign exchange data, jump detection, long memory, periodicity.

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1 Introduction

Price jumps and the periodic behavior of intraday volatility due to opening, lunch and closing of financial markets are salient features of high-frequency price series. Andersen and Bollerslev (1997b) and Andersen et al (2007a) document the importance of allowing for jumps and for periodicity of intraday volatility in the non-parametric estimation and forecasting of volatility, but treat these two features separately. Periodicity estimates that are robust to price jumps are needed for disentangling the periodic variation in volatility and the impact of news on intraday volatility (Dominguez and Panthaki, 2006), forecasting of intraday variance (Martens et al., 2002), efficient estimation of daily volatility using high-frequency returns (Areal and Taylor, 2002; Bos, 2008) and, as we show in this paper, for intraday jump detection.

Neglecting the potential presence of jumps when estimating the periodic component of intraday volatility can cause a large estimation bias. Andersen et al. (2001) find that removing one observation from the sample completely changes the estimated volatility pattern reported by Ito et al. (1998). Andersen et al. (2007b) note that jump detection based on comparing standardized intraday returns with critical values of the normal distribution "will tend to over-reject the diffusive null hypothesis whenever there is substantial intraday variation in volatility".

The contribution of this paper is twofold. First of all, we propose in Section 2 estimators for the periodic component in intraday volatility that are robust to price jumps. These estimators are robustifications of Andersen and Bollerslev (1997b) and Taylor and Xu (1997)'s periodicity estimators. Our second contribution is to show in Section 3 that the robust periodicity estimate can be used to increase the accuracy of the non-parametric intraday jump detection methods proposed by Andersen et al. (2007b) and Lee and Mykland (2008). Sections 2 and 3 conclude with an application of the methods to the 5-minute EUR/USD, GBP/USD and

YEN/USD returns. Section 4 summarizes our conclusions.

2 Robust estimation of intraweek periodicity

This section proposes robust estimators for the intraweek periodic pattern in the volatility of high-frequency return series. We begin with an empirical illustration to motivate the need for such methods.

For foreign exchange data, the monthly employment report on Friday 8:30 EST is directly associated to a large proportion of the price jumps in the data. This can be seen easily in Figure 1 where, on the left hand side, we report the boxplots of all 5-minute EUR/USD returns between 8:00 and 9:00 EST observed on Fridays in January 2001-December 2004 and on the right hand side only for those Fridays without employment report. The returns are standardized by a robust estimate of the average level of daily volatility.[1] In both boxplots we see that the boxplots first widen between 8:00 and 8:30 EST and then shrink again. There is a large number of outliers in the returns for the 8:30-8:35 EST interval. These outlying returns correspond to jumps in the price series. Comparing the boxplots in the left and right panel of Figure 1, we see that most of these outlying returns occur on days with a scheduled release of the employment report.

In the bottom panel of Figure 1 we plot the non-parametric periodicity estimator of Taylor and Xu (1997) for all 5-minute intervals on Friday. Note that the concentration of large price jumps between 8:30-8:35 EST leads to a sharp increase in the periodicity estimator and that this peak reduces by almost forty per cent if the days with employment report release are removed from the sample. The sharp increase in the Taylor-Xu standard deviation based periodicity estimate is thus due to the

[1] More details on the data are given in Subsection 2.4. The exact definition of the robust volatility estimate and Taylor-Xu periodicity estimate are given in Subsection 2.1.

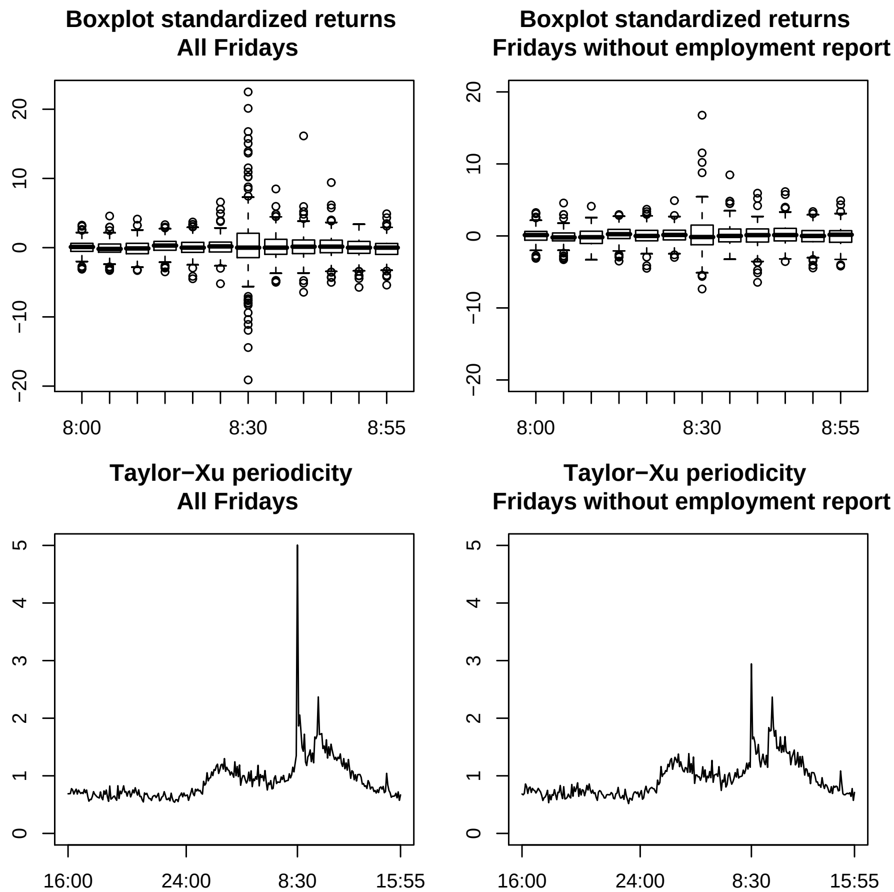

Figure 1: Illustration of the need for robust estimates of periodicity for 5-minute EUR/USD returns observed on Fridays. The returns are standardized by a robust estimate of the average level of daily volatility.

outlying returns.

Clearly, conditioning on the days without any news, would lead to a too small sample. Since the employment report has a monthly frequency, removing the Fridays with a scheduled employment report release already leads to removing 25% of the Friday returns in the data. Conditioning on more types of macroeconomic news releases would reduce the estimation sample further and still provides no guarantee that the final data set will be outlier-free. Finally, it should be emphasized that it is not the event of releasing news that explains price changes, but the surprise content of the news release. For practical estimation of the periodicity in intraday volatility a robust estimator is thus needed. In the next subsections such an estimator is developed.

We suppose that our estimation sample consists of T days of M equally-spaced and continuously compounded intraday return observations r_i ($i = 1, \ldots, MT$) of a financial asset. As usual when dealing with intraday data, we exclude overnight returns from the analysis. We normalize the length of one trading day to unity such that $\Delta = 1/M$ equals the time elapsed between two consecutive return observations. Hence, r_i equals the return over the time interval $[(i-1)\Delta, i\Delta]$.

Our main interest is in the estimation of the conditional standard deviation σ_i of the high-frequency return r_i. This is a difficult quantity to estimate and therefore usually more structure is imposed on the process. In the seminal papers of Andersen and Bollerslev (1997b) and Andersen and Bollerslev (1998b), the authors implicitly assume that the return r_i is a normal random variable with zero mean and that the standard deviation σ_i can be rewritten as the product between a deterministic component f_i and an average volatility factor which is constant in a "local window" around r_i. This yields the following data generating process for the high-frequency return

$$r_i = f_i s_i u_i, \tag{2.1}$$

where $u_i \overset{i.i.d.}{\sim} N(0,1)$. The factor f_i is supposed to be a deterministic function of periodic variables such as the time of the day, the day of the week and macroeconomic news announcements. The decomposition $\sigma_i = f_i s_i$, with s_i constant over the local window, is unique under the standardization condition that the squared periodicity factor has mean one over a local window. Similar standardization conditions have been considered in the literature, but ours has the advantage that under model (2.1) the periodicity pattern has no influence on the total variance over the local window.[2]

The value and the interpretation of the factor f_i depend thus on the choice of local window. In this paper the local windows are obtained by a division of $[0, T]$ in time intervals of length λ.[3] As such, the MT observations are divided in groups of $\lfloor \lambda/\Delta \rfloor$ contiguous observations. Denote by N_i the collection of indices j that belong to the same window as i. The standardization condition that the squared periodicity factor has mean one over the local window thus implies that for all $i = 1, \ldots, TM$

$$\frac{1}{\lfloor \lambda/\Delta \rfloor} \sum_{j \in N_i} f_j^2 = 1. \tag{2.2}$$

For descriptive studies of periodicity in intraday volatility of exchange rate data such as the 5-minute EUR/USD returns, it is common to set λ to one day (Andersen and Bollerslev, 1997b, 1998b). For the intraday jump tests based on the absolute return divided by a robust estimate of the scale of the returns in a local window of length λ, the choice of λ determines the accuracy of the local scale estimate. This choice is based on the trade-off that on the one hand we need a sufficiently large number of observations in the local window and on the other hand we need that, over the local window, the change in the (periodicity adjusted) volatility is negligible. While for intraday jump detection using tick by tick data of a liquid financial asset λ can be as small as 15 minutes, intraday jump detection using 5 or 15 minute returns requires a λ of one day (see e.g. Andersen et al., 2007b, and Lee and Mykland, 2008).

The returns in (2.1) can be seen as discrete changes of the underlying continuous-

[2]Taylor and Xu (1997) impose that the mean squared periodicity factor equals one over the whole cycle and not only over the local window. Andersen and Bollerslev (1997b) impose that the mean of the periodicity factor (and not the squared periodicity factor) equals one over the day.

[3]Our definition of windows is slightly different than in Lee and Mykland (2008), where the window equals the time period of length λ that immediately precedes the return for which the presence of jumps is tested. The exact definition of the window does not matter as long as it is reasonable to assume that σ_i^2/f_i^2 is approximately constant within the local window.

time log-price process, i.e. $r_i = p(i\Delta) - p((i-1)\Delta)$. Model (2.1) is motivated by the idea that this log-price process follows a *Brownian SemiMartingale* (BSM) diffusion. Under the BSM model the log-price follows a diffusion consisting of the sum of a conditionally normal random process with mean $\mu(s)ds$ and variance $\sigma^2(s)ds$. Let $w(s)$ be a standard Brownian motion, then a BSM log-price diffusion admits the following representation

$$\text{BSM:} \quad dp(s) \;=\; \mu(s)ds + \sigma(s)dw(s). \tag{2.3}$$

Throughout, we will be operating with sufficiently high-frequency return series such that the mean process can be safely ignored. This explains why in (2.1) the mean return is set to zero. The spot volatility $\sigma(s)$ has typically a slowly time-varying part and a periodic possibly rapidly varying component. The first component is modeled in (2.1) by the s_i which is assumed to be constant over the local window. The second component corresponds to the periodicity factor f_i in (2.1).

A more realistic representation of the data is that the log-price process $p(s)$ follows a *Brownian SemiMartingale with Finite Activity Jumps* (BSMFAJ) diffusion

$$\text{BSMFAJ:} \quad dp(s) \;=\; \mu(s)ds + \sigma(s)dw(s) + \kappa(s)dq(s), \tag{2.4}$$

where the occurrence of jumps is governed by a finite activity counting process $q(s)$ and the size of the jumps is given by $\kappa(s)$ (see e.g. Andersen et al., 2007b, and Lee and Mykland, 2008).[4] This suggests the following discrete time model

$$r_i = f_i s_i u_i + a_i, \tag{2.5}$$

[4] A count process is defined to be of finite activity if the change in the count process over any interval of time is finite with probability one.

where f_i, s_i and u_i are as in (2.1) and where a_i is a random variable that is zero for most of the observations. For the intervals $[(i-1)\Delta, i\Delta]$ in which jumps occur, a_i is non-zero and can be seen as an additive outlier with respect to $f_i s_i u_i$.

We use this very simple model to robustly estimate the deterministic component in the volatility of the returns standardized with a robust estimate of the average volatility of the r_j's belonging to the same local window as r_i. For this standardization we use the square root of a normalized version of Barndorff-Nielsen and Shephard (2004)'s realized bipower variation over the local window

$$\hat{s}_i = \sqrt{\frac{\pi}{2}\frac{1}{\lfloor\lambda/\Delta\rfloor - 1}\sum_{l=j+2}^{j+\lfloor\lambda/\Delta\rfloor} |r_l||r_{l-1}|}, \qquad (2.6)$$

where $r_{j+1}, \ldots, r_{j+\lfloor\lambda/\Delta\rfloor}$ are the $\lfloor\lambda/\Delta\rfloor$ returns in a local window of length λ around r_i.

The estimation of the periodic component in intraday volatility is based on the standardized high-frequency return

$$\overline{r}_i = r_i/\hat{s}_i. \qquad (2.7)$$

We have that under model (2.5) and in the absence of jumps in the interval $[(i-1)\Delta, i\Delta]$, $\overline{r}_i$ is asymptotically normally distributed with mean zero and variance equal to the squared periodicity factor. This result suggests to estimate the periodicity factor using either a non-parametric or parametric estimator of the scale of the standardized returns $\overline{r}_i$. Such an estimator has to be robust to price jumps.

2.1 Non-parametric estimation of periodicity

The non-parametric periodicity estimator is based on a scale estimate of the standardized returns that share the same periodicity factor. Let $\overline{r}_{1,i}, \ldots, \overline{r}_{n_i,i}$ be the set of standardized returns having the same periodicity factor as $\overline{r}_i$. If the periodicity factor depends only on the time of the day and day of the week at which r_i is observed, we have that $\overline{r}_{1,i}, \ldots, \overline{r}_{n_i,i}$ are the returns observed on the same time of the day and day of the week as r_i.

The non-parametric periodicity estimator proposed by Taylor and Xu (1997) is based on the Standard Deviation (SD) of all standardized returns belonging to the same local window as $\overline{r}_i$, i.e. $\text{SD}_i = \sqrt{\frac{1}{n_i}\sum_{j=1}^{n_i} \overline{r}_{j,i}^2}$. The SD periodicity estimator equals

$$\hat{f}_i^{\text{SD}} = \frac{\text{SD}_i}{\sqrt{\frac{1}{\lfloor \lambda/\Delta \rfloor}\sum_{j \in N_i} \text{SD}_j^2}}. \tag{2.8}$$

The denominator in (2.8) ensures that the standardization condition in (2.2) is met.

In the absence of jumps, the SD is efficient since the standardized returns are normally distributed. In the presence of jumps, the SD estimator is strongly biased, since it suffices that one observation in the sample is affected by an arbitrarily large jump to make the periodicity estimate extremely large. Our proposal is to replace the standard deviation in (2.8) by a robust estimator.

Amongst the large number of robust scale estimators available in the literature, we recommend the use of the Shortest Half scale estimator proposed by Rousseeuw and Leroy (1988). It has the property of being, among a wide class of scale estimators, the estimator for which jumps can cause the smallest maximum bias possible (Martin and Zamar, 1993). For the definition of the Shortest Half (ShortH) scale estimator, we need the corresponding order statistics $\overline{r}_{(1),i}, \ldots, \overline{r}_{(n_i),i}$ such that $\overline{r}_{(1),i} \leq \overline{r}_{(2),i} \leq \ldots \leq \overline{r}_{(n_i),i}$. The shortest half scale is the smallest length of all "halves" consisting of $h_i = \lfloor n_i/2 \rfloor + 1$ contiguous order statistics:

$$\text{ShortH}_i = 0.741 \cdot \min\{\overline{r}_{(h_i),i} - \overline{r}_{(1),i}, \ldots, \overline{r}_{(n_i),i} - \overline{r}_{(n_i-h_i+1),i}\}. \tag{2.9}$$

Analogous to the SD estimator in (2.8), the ShortH estimator for the periodicity factor of r_i equals

$$\hat{f}_i^{\text{ShortH}} = \frac{\text{ShortH}_i}{\sqrt{\frac{1}{\lfloor \lambda/\Delta \rfloor} \sum_{j \in N_i} \text{ShortH}_j^2}}. \tag{2.10}$$

The ShortH is highly robust to jumps, but it has only a 37% efficiency under normality of the $\overline{r}_i$'s (Rousseeuw and Leroy, 1988). A more efficient estimator than the ShortH, having also a high robustness to jumps, is obtained using the Weighted Standard Deviation (WSD), where the weights depend on the value of the standardized return divided by the ShortH periodicity estimate

$$\hat{f}_i^{\text{WSD}} = \frac{\text{WSD}_i}{\sqrt{\frac{1}{\lfloor \lambda/\Delta \rfloor} \sum_{j \in N_i} \text{WSD}_j^2}}, \tag{2.11}$$

with

$$\text{WSD}_j = \sqrt{1.081 \cdot \frac{\sum_{l=1}^{n_j} w_{l,j} \overline{r}_{l,j}^2}{\sum_{l=1}^{n_j} w_{l,j}}}. \tag{2.12}$$

The weights are given by $w_{l,j} = w(\overline{r}_{l,j}/\hat{f}_j^{\text{ShortH}})$ where we use as a weight function $w(z) = 1$ if $z^2 \leq 6.635$ and 0 otherwise. The threshold 6.635 equals the 99% quantile of the χ^2 distribution with one degree of freedom. If there are no price jumps, the WSD gives a zero weight to on average 1% of the returns. If there are jumps, more observations are downweighted. The WSD in (2.12) has a 69% efficiency under normality of the $\overline{r}_i$'s, as opposed to the 37% efficiency of the ShortH (see Boudt et al., 2008, for details).

2.2 Parametric estimation of periodicity

The non-parametric estimators for the periodic component of intraday volatility use only the subset of the data for which the returns have the same periodicity factor. Andersen and Bollerslev (1997b) show that more efficient estimates can be obtained if the whole time series dimension of the data is used for the estimation of the periodicity process. They use the result that, in the absence of jumps, the standardized returns are normally distributed with mean zero and variance f_i^2. They consider the regression equation

$$\log |\overline{r}_i| - c = \log f_i + \varepsilon_i, \tag{2.13}$$

where the error term ε_i is i.i.d. distributed with mean zero and having the density function of the centered absolute value of the log of a standard normal random variable, i.e.

$$g(z) = \sqrt{2/\pi}\exp[z + c - 0.5\exp(2(z + c))]. \tag{2.14}$$

The parameter $c = -0.63518$ equals the mean of the log of the absolute value of a standard normal random variable. Andersen and Bollerslev (1997b) then propose to model $\log f_i$ as a linear function of a vector of variables x_i (such as sinusoid and polynomial transformations of the time of the day)

$$\log f_i = x_i'\theta_*, \tag{2.15}$$

with θ_* the true parameter value. Combining (2.13) with (2.15), we obtain the following regression equation

$$\log |\overline{r}_i| - c = x_i'\theta_* + \varepsilon_i. \tag{2.16}$$

The regression specification used in the empirical application of this paper is given in Appendix A.

It is common to estimate the parameter θ_* in (2.16) by OLS. However, this approach is not efficient, since the error term is not normally distributed. The efficient estimator is the maximum likelihood estimator. Denote $\rho^{\text{OLS}}(z) = z^2$ and let $\rho^{\text{ML}}(z)$ be the negative log likelihood function

$$\rho^{\text{ML}}(z) = -0.5\log(2/\pi) - z - c + 0.5\exp(2(z + c)).$$

The OLS and ML estimates are given by

$$\hat{\theta}^{\text{OLS}} = \text{argmin}_{\theta}\frac{1}{MT}\sum_{i=1}^{MT}\rho^{\text{OLS}}(\varepsilon_{i,\theta}) \quad \text{and} \quad \hat{\theta}^{\text{ML}} = \text{argmin}_{\theta}\frac{1}{MT}\sum_{i=1}^{MT}\rho^{\text{ML}}(\varepsilon_{i,\theta}), \quad (2.17)$$

with $\varepsilon_{i,\theta} = \log|\overline{r}_i| - c - x_i'\theta$. These ρ-functions are called loss functions. The non-robustness of the OLS and ML estimators to jumps is due to the unbounded effect an observation can have on their loss function. In the simulation study of Subsection 2.3 we find that in particular the ML estimator has a large bias in the presence of jumps. Martens et al. (2002) mention that the effect of jumps on the OLS estimator is attenuated because the regression is based on the log of the standardized returns, but solely a log-transformation is not sufficient to attain robustness to jumps.

As an alternative to the OLS and ML estimators, we propose to use the *Truncated Maximum Likelihood* (TML) estimator introduced by Marazzi and Yohai (2004). This estimator gives a zero weight to observations that are outliers according to the value of the ML loss function. In a first step the residuals are computed using the robust non-parametric estimator $\hat{f}^{\text{WSD}}$ in (2.11). Let

$$e_i^{\text{WSD}} = \log|\overline{r}_i| - c - \log\hat{f}_i^{\text{WSD}}. \quad (2.18)$$

Observations for which $\rho^{\mathrm{ML}}(e_i^{\mathrm{WSD}})$ is large, have a low likelihood and are therefore likely to be outliers. Denote q an extreme upper quantile of the distribution of ε_i. The TML estimator is defined as

$$\hat{\theta}^{\mathrm{TML}} = \mathrm{argmin}_{\theta} \frac{1}{\sum_{i=1}^{MT} w_i} \sum_{i=1}^{MT} w_i \rho^{\mathrm{ML}}(\varepsilon_{i,\theta}), \tag{2.19}$$

with $w_i = 1$ if $\rho^{\mathrm{ML}}(e_i^{\mathrm{WSD}}) \leq \rho^{\mathrm{ML}}(q)$ and 0 otherwise. Henceforth, we take q as the 99.5% quantile such that all observations with $\rho^{\mathrm{ML}}(e_i^{\mathrm{WSD}}) > 3.36$ receive a zero weight in the objective function of the TML estimator. Like for the WSD, the choice of these thresholds implies that, if there are no price jumps, the TML gives a zero weight to on average 1% of the returns. If there are jumps, more observations are downweighted.

Like for the non-parametric periodicity estimators, we impose that the squared periodicity factor has mean one in the local window. The parametric estimate for the periodicity factor thus equals

$$\hat{f}_i^{\mathrm{TML}} = \frac{\exp x_i' \hat{\theta}^{\mathrm{TML}}}{\sqrt{\frac{1}{\lfloor \lambda/\Delta \rfloor} \sum_{j \in N_i} (\exp x_i' \hat{\theta}^{\mathrm{TML}})^2}}, \tag{2.20}$$

and similarly for $\hat{f}_i^{\mathrm{OLS}}$ and $\hat{f}_i^{\mathrm{ML}}$.

2.3 Simulation study

In this section we use simulated data to evaluate the effect of jumps on the bias and efficiency of the periodicity estimators to jumps. Let $w(s)$ and $b(s)$ be two independent Brownian motions. We generate 5-minute returns from the BSMFAJ price diffusion in (2.4) with $\mu(s) = 0$ and $\sigma(s)$ specified as a multiplicative process of the periodicity function $f(\tau(s))$, which depends only on the time of the day

$\tau(s) = s - \lfloor s \rfloor$, and a GARCH diffusion process, i.e.

$$\sigma(s) = f(s)\sigma_{\text{garch}}(s). \tag{2.21}$$

The GARCH diffusion is calibrated as in Andersen and Bollerslev (1998a)

$$d\sigma^2_{\text{garch}}(s) = -.035[\sigma^2_{\text{garch}}(s) - .636]ds + .144\sigma^2_{\text{garch}}(s)db(s). \tag{2.22}$$

The periodicity function $f(s)$ is modeled as a function of the intraday time. It has been calibrated at its TML estimate for the January 2001 - December 2004 5-minute EUR/USD returns. It is plotted in dashed line in Figure 2. The jump size $\kappa(s)$ is modeled as the product between a uniformly distributed random variable on $\sqrt{m}([-2,-1] \cup [1,2])$ and total spot volatility $\sigma(s)$. The parameter m determines the magnitude of the jumps. We set m equal to either 0.1 (small jumps) or 1 (large jumps). For $m = 0.1$, jumps cause about 20% of the daily variance of the returns. Finally, the jump occurrences $q(s)$ are specified as a Poisson process with on average one jump per day. These jump occurrences are either independent of f, either occur only in the 16 five-minute intervals for which volatility is periodically the lowest ($f < 0.777$) or in the 16 five-minute intervals for which volatility is periodically the highest ($f > 1.3$).

We simulate $K = 500$ series of 500 days with 10 observations per 5-minute interval. Each day consists of 288 5-minute returns. The generated 5-minute return series is $r_i = p(i/288) - p((i-1)/288)$, with $i = 1, \ldots, 288 \cdot 500$. The estimation of the periodicity factor is based on a local window length λ equal to one day.

We are interested in the effect of jumps on the bias and efficiency of the periodicity estimators. Recall that the non-parametric estimators use either the SD, ShortH or WSD as a scale estimator and that under the parametric approach, we

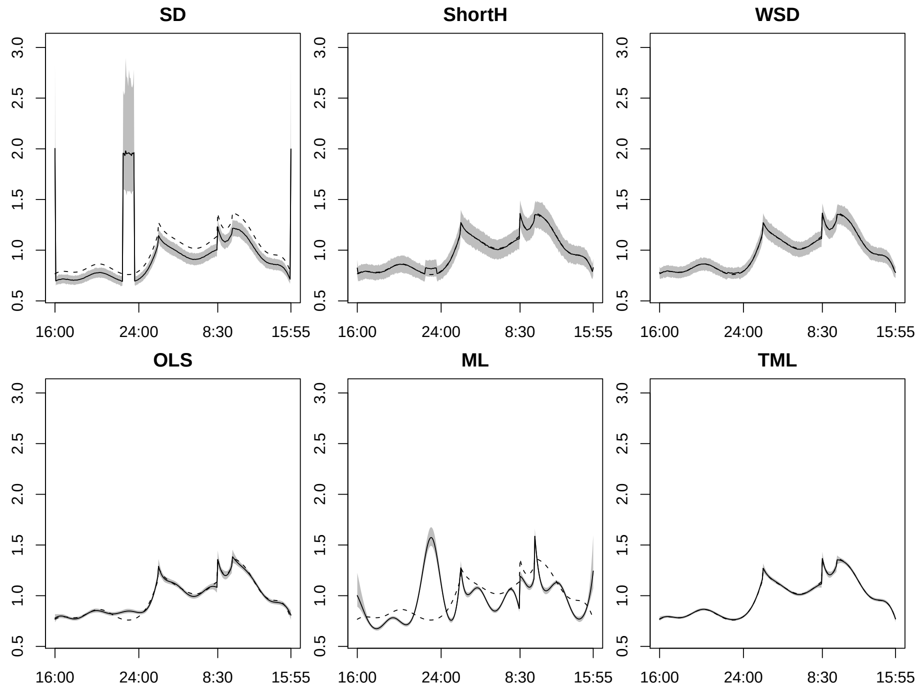

Figure 2: True periodicity function (dashed lines) versus the average estimate across the simulations (full line), for 6 different estimation procedures. The shaded region is the range between the 2.5% and 97.5% quantiles. All jumps occur when $f < 0.777$ and jumps are small ($m = 0.1$).

have the choice between the OLS, ML or TML parameter estimators. This yields a total of 6 estimation methods.

Bias. Figure 2 compares the true periodicity function f with the average estimate across the 500 simulations. The difference between both indicates the bias of the periodicity estimators caused by the price jumps. Jumps are small ($m = 0.1$) and their occurrences are concentrated on the intraday times for which $f < 0.777$. We see that the SD, ML and OLS estimators overestimate the periodic component for the intervals with jumps and underestimate it for the intervals without jumps. The WSD and TML estimators are the only estimators without visual evidence of a bias.

Efficiency. In Figure 2 we also report the 95% confidence bands. We see that, in the presence of jumps, the SD estimator has the largest standard error. More details

Table 1: RMSE of estimators for the periodicity factor. Either no jumps or one jump per day. Jumps are small ($m = 0.1$) or large ($m = 1$) and their occurrences are either uniformly distributed over the day or concentrated on the parts of the day when volatility is periodically low ($f < 0.777$) or high ($f > 1.333$).

	non-parametric estimation			parametric estimation		
RMSE	SD	ShortH	WSD	OLS	ML	TML
No jumps	0.032	0.049	0.038	0.014	0.009	0.010
$m = 0.1$, uniform	0.080	0.050	0.038	0.014	0.022	0.010
$m = 1$, uniform	0.269	0.050	0.037	0.015	0.076	0.010
$m = 0.1$, $f < 0.777$	0.307	0.052	0.038	0.029	0.229	0.010
$m = 1$, $f > 1.3$	0.223	0.055	0.038	0.043	0.204	0.010

on the relative efficiency of the estimators in the absence and presence of jumps are given in Table 1. It reports the root mean squared error (RMSE) of the estimators across the 500 simulations.

The first row in Table 1 reports the RMSE of the estimators if there are no price jumps. We see that all parametric estimators have a much lower RMSE than the non-parametric estimators. In the class of non-parametric estimators, the SD is the most efficient estimator. The ShortH has the largest RMSE and the RMSE of the WSD is between the RMSE of the SD and ShortH. For the parametric estimators, the ML estimator is efficient. The TML estimator is only slightly less efficient than the ML estimator and the OLS estimator has the largest RMSE of all parametric estimators.

Rows 2-5 of Table 1 report the RMSE for a log-price diffusion with on average one jump per day. We see that the RMSE of the SD in the presence of jumps is several times higher than its RMSE in the absence of jumps. Also the ML estimator is extremely sensitive to jumps. The RMSE of the ShortH, WSD, OLS and TML periodicity estimators is little affected by the inclusion of jumps in the price process. The robustness of the OLS estimator is surprising at first sight, but it corroborates

Martens et al. (2002)'s intuition that the log-transformation shrinks the outliers and makes the estimators based on a regression of the log absolute returns more robust to jumps. Note, however, that the TML estimator has a significantly lower RMSE than the OLS estimator in all simulations considered here.

The main message of Table 1 is that the non-parametric WSD and parametric TML estimators have a relatively high efficiency in the absence of jumps. If jumps are present in the process, they are the most accurate of all non-parametric and parametric estimators considered, respectively. Because of its high efficiency, we recommend to estimate sufficiently smooth periodicity functions using the TML estimator based on a flexible parametric specification of the periodicity function.

2.4 Empirical results

The Monte Carlo study has shown that jumps induce a large bias in the classical periodicity estimate, especially if the jump occurrences are also periodic. But how different are the periodicity plots when estimated on real data? Our data set, provided by Olsen and Associates, ranges from January 2001 - September 2004 for the EUR/USD returns and from January 2003-September 2004 for the GBP/USD and YEN/USD returns. Time is expressed in Eastern Standard Time (EST), taking into account the daylight saving time shifts in Europe and America. The raw data consists of last mid-quotes of 5-minute intervals throughout the global 24-hour trading day. We obtain 5-min returns as 100 times the first difference of the logarithmic prices. Weekends are excluded. One trading day extends from 16.00 EST on date $t-1$ to 16.00 EST on date t. We consider a local window length λ equal to one day and thus assume that the intraday variation in volatility is mainly due to a periodic function f that is a deterministic function of the intraweek time.[5]

[5] An alternative approach is to condition not only on intraweek time, but also on macroeconomic news announcements (Andersen and Bollerslev, 1997b) or to allow the periodicity in intraday

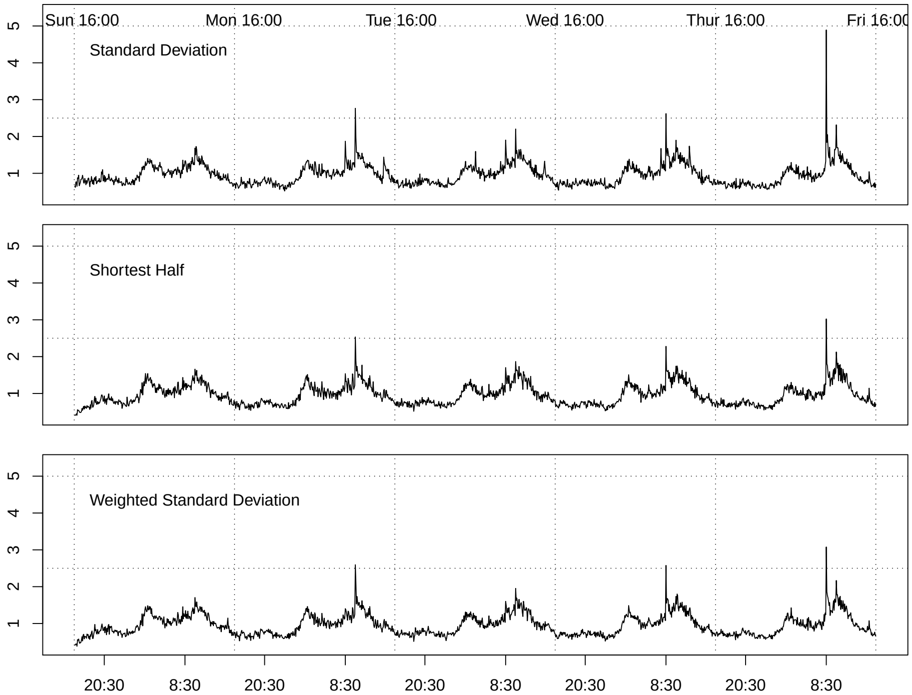

Figure 3: Non-parametric SD, ShortH and WSD estimates of the intraweek periodicity in the volatility of 5-minute EUR/USD returns. Time is expressed in EST.

Figure 3 plots the SD, ShortH and WSD-based non-parametric estimates of the intraweek periodic component in the volatility of 5-minute EUR/USD returns. For the parametric periodicity estimates in Figure 4 we use a regression model that is similar to the one proposed by Andersen and Bollerslev (1997b). It specifies the log of the intraweek periodicity factor as a function of the time of the day interval and day of the week. The regression specification is given in Appendix A.

The first value in these plots is the periodicity factor for the 5-minute return on Sunday evening at 16:30-16:35 EST. We see that for most intervals, the classical and robust periodicity estimates closely resemble each other. Two notable exceptions are the 8:30-35 and 10:00-10:05 EST intervals during which the most important macroeconomic news are released and Sunday evening. The Sunday 16:30-16:35 EST

volatility to be stochastic (Beltratti and Morana, 2001).

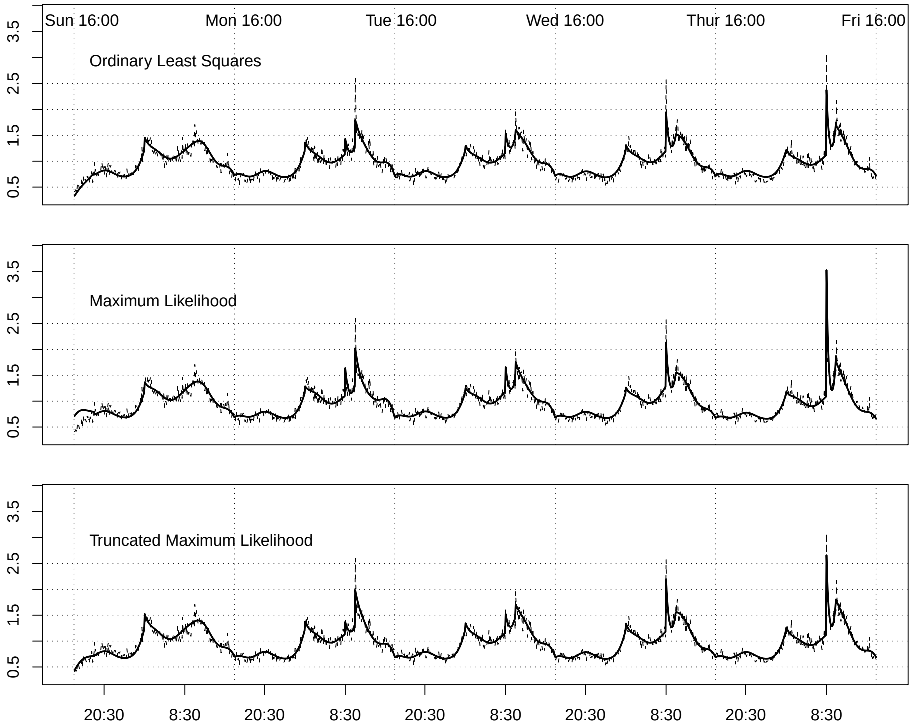

Figure 4: Parametric OLS, ML and TML estimates of the intraweek periodicity in the volatility of 5-minute EUR/USD returns. The WSD estimates (dashed line) are reported as benchmark. Time is expressed in EST.

periodicity factor equals 0.652 according to the SD estimator and only 0.416 according to the WSD estimator. Consider now the 8:30-8:35 interval on Friday morning. According to the SD estimator, the periodicity factor is around 5, while according to the robust non-parametric estimators and all parametric estimators it is between 2.5 and 3.5. Such a large difference between these estimators can only be due to the presence of jumps in the data. This mirrors the fact that in this interval many macroeconomic news are released and that these news releases are often associated with jumps. Note however that also according to the robust estimators, there is a sharp increase in the periodic component of intraday volatility at the time of the macroeconomic news releases.[6] This result is consistent with the finding of Jiang

[6]Macro-economic news releases often have a monthly frequency. Since the release of different types of news are spread over the different weeks of the month (see e.g. Table 2 in Andersen et al.,

et al. (2009) that also on days without jumps, the return volatility on the U.S. Treasury market starts to rise in the 5-minute interval before an announcement and then peaks at the announcement time. A similar result is reported by Chaboud et al. (2008) for liquidity data. These authors conclude that "scheduled U.S. macroeconomic announcements, in particular, are clearly associated with spikes in trading volume but these spikes tend to occur even if the announcements are in line with market expectations (and therefore generate little price response)".

Figure 4 compares the parametric periodicity estimates with the WSD estimate. We see that, on Sunday evening, the OLS and TML periodicity estimators are close to the WSD estimate, and that the ML estimate is completely different than the robust estimates. The peaks in the OLS and TML parameter estimates are always smaller than the peaks in the WSD estimate.

Periodicity estimates have been used for disentangling the periodic variation in volatility and the impact of news on intraday volatility (Andersen et al., 2003; Dominguez and Panthaki, 2006), forecasting of intraday variance (Martens et al., 2002) and efficient estimation of daily volatility using high-frequency returns (Areal and Taylor, 2002; Bos, 2008). For all these applications, it is natural to use a jump-robust periodicity estimate. In the remainder of the paper, we develop a framework of using jump-robust periodicity estimates for intraday jump detection.

3 Intraday jump tests

3.1 The original test

Andersen et al. (2007b) and Lee and Mykland (2008) use the absolute value of the standardized return $\overline{r}_i = r_i/\hat{s}_i$ in (2.7) as a test statistic for the null hypothesis that

2003), the release of a news on Friday 8:30 has a weekly frequency and therefore leads to a sharp increase in the periodic component of intraday volatility of the Friday 8:30-8:35 return.

r_i is not affected by jumps. Denote the jump test statistic for r_i by

$$\mathrm{J}_i = \frac{|r_i|}{\hat{s}_i}. \tag{3.1}$$

If r_i is not affected by a jump, then under model (2.5) the statistic J_i has approximately the same distribution as the absolute value of a standard normal random variable (see Theorem 1 in Lee and Mykland, 2008).

A straightforward jump detection rule is that return r_i is affected by a jump if J_i exceeds the $1-\alpha/2$ quantile of the standard Gaussian distribution. This rule has a probability of type I error (detect that r_i is affected by jumps, if in reality r_i is not affected by jumps) equal to α. But its disadvantage is that the expected number of false positives over the whole estimation sample becomes large. For example, with $M = 288$ intraday returns per day and $\alpha = 0.01$, one expects to detect about $0.01 \cdot 288 \approx 3$ jumps per day, even if no single jump has occurred. Lee and Mykland (2008) call these false positives "spurious jump detections".

Andersen et al. (2007b) use a Bonferroni correction to control for the number of spurious jumps detected per day. As a rejection threshold, they propose to use the $[1-(1-\alpha)^{\Delta}]/2$ quantile of the Gaussian distribution. Lee and Mykland (2008) control for the size of the multiple jump tests using the extreme value theory result that the maximum of n i.i.d. realizations of the absolute value of a standard normal random variable is asymptotically (for $n \to \infty$) Gumbel distributed. More specifically, in the absence of jumps, the probability that the maximum of any set of n J-statistics exceeds

$$g_{n,\alpha} = -\log(-\log(1-\alpha))b_n + c_n, \tag{3.2}$$

with $b_n = 1/\sqrt{2\log n}$ and $c_n = (2\log n)^{1/2} - [\log \pi + \log(\log n)]/[2(2\log n)^{1/2}]$, is about α. Lee and Mykland (2008)'s proposal is that all returns for which the J test statistic exceeds this threshold $g_{n,\alpha}$ should be declared as being affected by jumps. In the sequel of the paper, we use $n = \lfloor 1/\Delta \rfloor = 288$. This corresponds to testing for the joint null hypothesis of no jumps over one day. We set $\alpha = 1\%$. For these values of n, Δ and α, Andersen et al. (2007b) 's and Lee and Mykland (2008)'s threshold equals 4.139 and 4.305, respectively.

3.2 The filtered test

The original test in (3.1) assumes that the spot volatility $\sigma(s)$ is approximately constant over the local window used to compute $\hat{s}_i$ in (2.6). This is a reasonable assumption for short local windows such as 30 minutes. However, if returns are sampled at frequencies of one hour, 30 minutes, 15 minutes or 5 minutes, Lee and Mykland (2008) recommend to use local windows containing 78, 110, 156 or 270 observations, respectively.[7] This corresponds to local windows of at least 90% of a day. Also Andersen et al. (2007b) use local windows of one day in their application on the 2-min transaction returns from the S&P 500 futures contract. For such long windows, the assumption of constant volatility is at odds with the overwhelming empirical evidence that the intraday variation in market activity causes intraday volatility to be strongly time-varying and even displays discontinuities (Taylor, 2004). Consequently $\hat{s}_i$ does not estimate the volatility of r_i, but the average level of volatility of the returns in the local window of r_i.

Andersen et al. (2007b) recognize this and as a robustness check, they verify their empirical results using the returns standardized by a periodicity estimate that is similar to the SD estimator. The test statistic based on the original J statistic divided by a periodicity estimate is called the "filtered J test statistic". Andersen

[7] These numbers correspond to the smallest number of observations for which jumps will have a negligible effect on the realized bipower variation based estimate of $\hat{s}_i$ in (2.6).

et al. (2007b) use the filtered J test statistic based on the SD periodicity estimator

$$\text{FJ}_i^{\text{SD}} = \frac{|r_i|}{\hat{f}_i^{\text{SD}} \hat{s}_i^{\text{SD}}}, \tag{3.3}$$

where $\hat{s}_i^{\text{SD}}$ is the realized bipower variation scale estimator in (2.6) computed on the SD filtered returns $(r_j/\hat{f}_j^{\text{SD}})$ in the local window around r_i. We propose to use a filtered J test based on the robust WSD and TML periodicity estimates, i.e.

$$\text{FJ}_i^{\text{WSD}} = \frac{|r_i|}{\hat{f}_i^{\text{WSD}} \hat{s}_i^{\text{WSD}}} \quad \text{and} \quad \text{FJ}_i^{\text{TML}} = \frac{|r_i|}{\hat{f}_i^{\text{TML}} \hat{s}_i^{\text{TML}}}, \tag{3.4}$$

where $\hat{s}_i^{\text{WSD}}$ and $\hat{s}_i^{\text{TML}}$ are the realized bipower variation scale estimator in (2.6) computed on the WSD and TML filtered returns in the local window around r_i.

3.3 Simulation study

We now compare testing for jumps using the original and filtered J test statistics by means of a simulation study. The implementation is based on a local window length λ equal to one day. We use the rejection threshold in (3.2) with $\alpha = 0.01$ and $n = 288$ (the number of 5-minute intervals per day of 24 hours). This means that the returns for which the J test statistic exceeds 4.305 are identified as being affected by jumps. We generate $K = 500$ series of 500 days of 5-minute returns from a process that is the same as in Subsection 2.3, except that the specification of the periodicity function is more simple, namely:

$$f(s) = 0.447 \cdot I(\tau(s) \leq 1/3) + I(1/3 < \tau(s) \leq 2/3) + 1.342 \cdot I(2/3 < \tau(s) \leq 1), \tag{3.5}$$

where $\tau(s) = s - \lfloor s \rfloor$ equals the intraday component of time s. Under this specification, the intraday volatility is periodically low in the first 8 hours of the day and periodically high in the last 8 hours of the day.

Like Andersen et al. (2007b) and Lee and Mykland (2008) we use the proportion of spuriously detected jumps and proportion of actual jumps that have been detected with success as indicators of the size and power of the test. Andersen et al. (2007b) call these statistics "effective size" and "effective power", respectively. They are reported in Table 2 for the case of no jumps and 1 jump per day. Jump occurrences are uniformly distributed over the day. Their size is proportional to the spot volatility or daily volatility, with factor $m = 0.1$ (small jump) or $m = 1$ (large jumps). The effective size and power are reported as a function of the value of the periodicity function.

Effective size. The first panel in Table 2 reports the proportion of returns for which a jump has been detected by the original and the filtered J tests, if in reality there are no jumps in the process. Recall that a jump is detected when the original or filtered J statistics exceed 4.305. The probability that the absolute value of a standard normal exceeds 4.305 is equal to 1.7e-5. Because we take λ equal to one day and because of the time-varying volatility the actual effective size is slightly higher. Note that the original J test has an important size distortion for $f \neq 1$. If $f = 0.447$, it detects no spurious jumps at all and if $f = 1.342$ then 0.14% of all returns are (spuriously) identified as being affected by jumps. The original J test thus underdetects (overdetects) jumps if the value of the periodicity function is low (high). The differences in effective size of the filtered J test are economically insignificant with respect to the variation in effective size observed for the J test. In the case of 500 days of 288 5-minute returns, the J statistic detects on average between 0 ($f = 0.447$) and 202 ($f = 1.342$) spurious jumps, while the filtered J tests detect only between 3 ($\mathrm{FJ}^{\mathrm{SD}}$; $f = 1.342$) and 7 ($\mathrm{FJ}^{\mathrm{WSD}}$; $f = 0.447$) spurious jumps.

In panels 2 and 3 of Table 2 we report the effective size of the tests in the

Table 2: Effective size and power for the original and filtered J tests with rejection threshold equal to 4.305 as a function of the periodicity factor. Jump occurrences are uniformly distributed over the day. Their size is proportional to the spot volatility or daily volatility, with factor $m = 0.1$ (small jumps) or $m = 1$ (large jumps).

	effective size				effective power			
	J	FJ^{SD}	FJ^{WSD}	FJ^{TML}	J	FJ^{SD}	FJ^{WSD}	FJ^{TML}
No jumps								
$f = 0.447$	0	1.9e-5	2.8e-5	2.3e-5				
$f = 1$	2.4e-5	1.7e-5	2.6e-5	2.1e-5				
$f = 1.342$	1.4e-3	2.2e-5	3.0e-5	2.5e-5				
One small jump per day and jump size proportional to daily volatility								
$f = 0.447$	0	4.0e-5	3.1e-5	2.5e-5	0.9966	0.9994	0.9996	0.9996
$f = 1$	3.1e-5	7.8e-5	2.8e-5	2.2e-5	0.9782	0.9780	0.9743	0.9755
$f = 1.342$	1.8e-3	1.2e-4	3.0e-5	2.5e-5	0.9583	0.8672	0.8121	0.8144
One large jump per day and jump size proportional to daily volatility								
$f = 0.447$	0	1.4e-3	2.0e-5	1.7e-5	0.9995	0.9996	0.9997	0.9997
$f = 1$	1.7e-5	1.5e-3	1.7e-5	1.3e-5	0.9997	0.9997	0.9995	0.9995
$f = 1.342$	1.3e-3	1.8e-3	2.1e-5	1.8e-5	0.9996	0.9995	0.9990	0.9990
One small jump per day and jump size proportional to spot volatility								
$f = 0.447$	0	3.6e-5	3.1e-5	2.5e-5	0.1841	0.9678	0.9762	0.9766
$f = 1$	2.6e-5	3.3e-5	2.6e-5	2.0e-5	0.9766	0.9703	0.9769	0.9779
$f = 1.342$	1.8e-3	4.1e-5	3.1e-5	2.5e-5	0.9974	0.9718	0.9771	0.9778
One large jump per day and jump size proportional to spot volatility								
$f = 0.447$	0	9.1e-4	2.3e-5	1.7e-5	0.9982	0.9997	0.9997	0.9997
$f = 1$	2.1e-5	9.4e-3	2.0e-5	1.6e-5	0.9996	0.9995	0.9996	0.9996
$f = 1.342$	1.4e-3	1.0e-3	2.2e-5	1.8e-5	0.9996	0.9995	0.9995	0.9995

presence of jumps. Note that also in the presence of jumps, the effective size of the J test is highly dependent on the value of the periodicity factor. For all types of jumps considered, the effective size of the filtered J test based on the WSD or TML estimators has the correct order of magnitude (1e-5). In the case of large jumps ($m = 1$), the effective size of the filtered J test based on the SD estimator is too large. It is about 1.8e-3, which implies on average 260 spurious jump detections per 500 days. This is due to the bias and inefficiency of the SD periodicity estimator in the presence of large jumps. This phenomenon whereby the nonregular observations (the jumps) cause nonoutliers to be identified as outliers is called swamping.

Effective power. The detection of large jumps is only marginally affected by the presence of periodicity in the spot volatility. Let us therefore focus on the power to detect small jumps, which are the most difficult to detect. In panel 2 of Table 2, the jump size is independent of the seasonality in the spot volatility. Because the filtered jump test controls the size of the test by dividing the J test with a robust estimate of the periodicity factor, it has a lower power to detect the small jumps at times of high periodicity (81%) and a slightly higher power to detect the small jumps at times of low periodicity. In panel 3 of Table 2, the jump size is proportional to the spot volatility, meaning that at times of high (low) intraday periodicity, the jump size tends to be higher (lower). We see that in this case the original J test detects only 18% of all jumps if jumps are small ($m = 0.1$) and occur when volatility is periodically low ($f = 0.447$). The robustly filtered J tests detect more than 96% of the actual jumps in this case.

In conclusion, the original J test and the filtered J test based on the standard deviation have a large size distortion. The robustly filtered J test has a higher power to detect small jumps at times of low periodicity and a smaller power at times of high periodicity.

3.4 Empirical results

In this subsection we show that for the EUR/USD, GBP/USD and YEN/USD 5-minute returns, accounting for the periodic pattern in intraday volatility reduces dramatically the number of jumps detected at times of periodically high volatility and leads to a significant increase in the number of jumps detected at times of periodically low volatility. The data set is the same as in Subsection 2.4. The filtered J test is implemented with our preferred TML periodicity estimator. For all three currencies, we find that both the J and the filtered J tests detect that around

0.4% of the returns are affected by jumps. But, as can be seen in Figure 5 where we plot the proportion of returns affected by jumps according to the original and the filtered J tests, they detect different returns as jumps. The use of the filtered J test leads to a more uniform distribution of the number of jumps detected over the week.

The most striking differences are for the Tuesday-Friday 8:30-8:35 EST and Monday-Friday 10:00-10:05 EST intervals for which the periodicity is the highest and the Sunday evening (16:00-19:00 EST) for which the periodicity factor is the lowest. According to the original J test 25% (resp. 26% and 23%) of all EUR/USD (resp. GBP/USD and YEN/USD) returns in the Friday 8:30-8:35 EST interval, are affected by jumps and for the filtered J test it is 9% (resp. 7% and 5%). The proportion of jumps detected in the 5-minute EUR/USD, GBP/USD and YEN/USD returns in the Sunday evening interval is 0.2%, 0.1% and 0.3% for the J test and 1%, 0.8% and 0.6% for the filtered J test. This result is expected, since the simulation study showed that, unlike the filtered J test, the J test, on the one hand, overdetects jumps at times of high periodicity and underdetects jumps at low periodicity times. On the other hand, the original J test has a higher (lower) power to detect small jumps at high (low) periodicity and inversely for the filtered J test.

The jump test statistic corrected for periodicity has several applications. Lahaye et al. (2009) study the relation between macroeconomic news and the jumps detected using the proposed jump test statistic. The application we consider here is to use the series of detected price jump occurrences, to construct a jump robust estimate of the autocorrelation and long-memory in the absolute or squared TML filtered high-frequency return series $(|r_1|/\hat{f}_i^{\text{TML}})^q, \ldots, (|r_{MT}|/\hat{f}_{MT}^{\text{TML}})^q$, with $q = 1, 2$.

Let $L_1, ..., L_{MT}$ be the series of dummy observations with value one if the TML test has not detected a price jump in the corresponding return. Write the series for

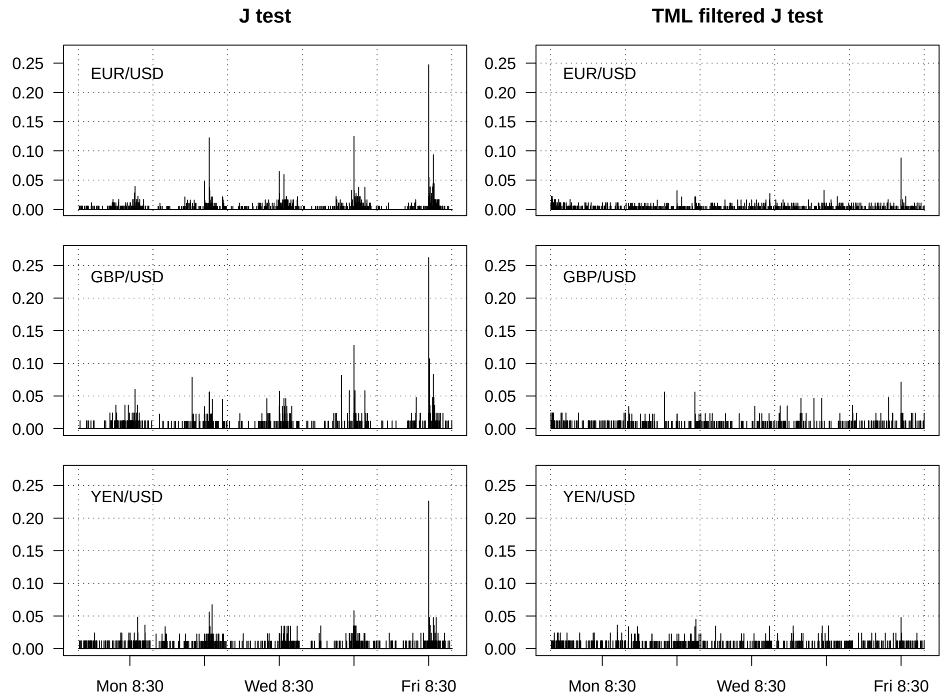

Figure 5: Proportion of 5-minute EUR/USD, GBP/USD and YEN/USD returns for which the original and truncated maximum likelihood filtered J test, with rejection threshold equal to 4.305, detect jumps.

which we estimate the autocorrelation as $y_1, \ldots, y_{MT}$. Let m and S be the mean and variance of all y_i for which there is no jump in the original return series, i.e.

$$m = \frac{1}{\sum_{i=1}^{MT} L_i} \sum_{i=1}^{MT} y_i L_i \quad \text{and} \quad S = \frac{1}{\sum_{i=1}^{MT} L_i} \sum_{i=1}^{MT} (y_i - m)^2 L_i.$$

A highly jump-robust estimate of the autocorrelation of order l in the y-series is

$$\hat{\rho}_l^{\mathrm{rob}} = \frac{1}{S} \frac{1}{\sum_{i=l+1}^{MT} L_i L_{i-l}} \sum_{i=l+1}^{MT} (y_i - m)(y_{i-l} - m) L_i L_{i-l}. \tag{3.6}$$

The proposed autocorrelation estimator is directly related to the trimmed autocorrelation estimator of Chan and Wei (1992). The classical and robust first order autocorrelation of the 5-min EUR/USD and GBP/USD series are similar (around

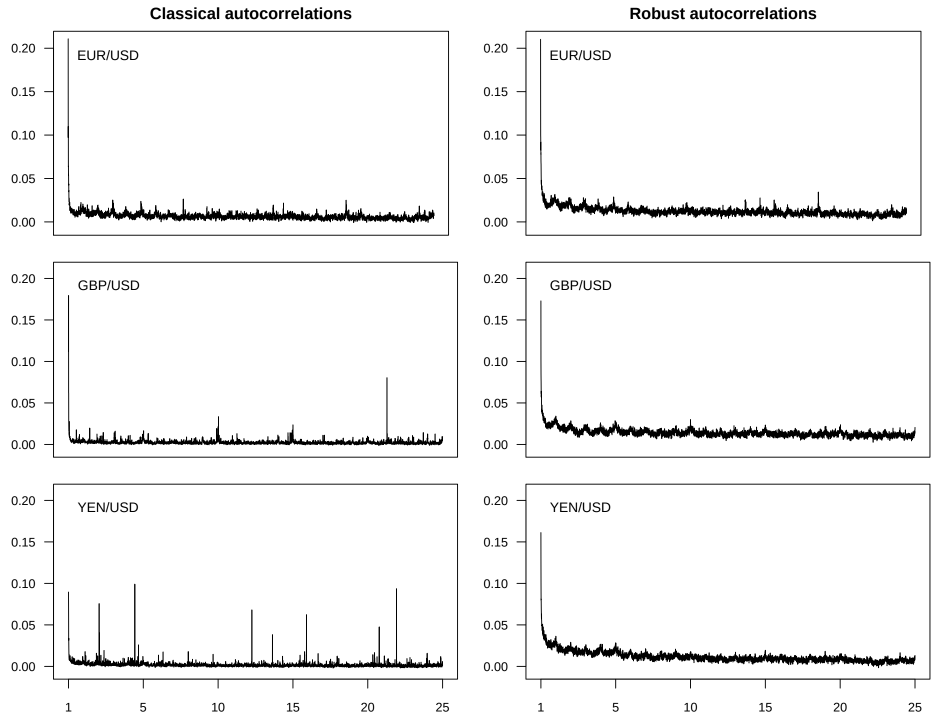

Figure 6: Classical and robust autocorrelation function of squared TML filtered 5-minute EUR/USD, GBP/USD and YEN/USD returns. The lags are expressed in number of days.

21% and 17% respectively), but the robust first order autocorrelation of the squared 5-minute YEN/USD return is 16% while the classical one is only 10%. The autocorrelation estimates are plotted in Figure 6. Note that for almost all lags the robust autocorrelations are higher than the classical ones and that the classical autocorrelation estimate in the GBP/USD and Yen/USD show rather large spikes. Making abstraction of these spikes, the classical and robust autocorrelation estimates display a slow and smooth decay for all three currencies.

Andersen and Bollerslev (1997a) develop a model that attributes the slow decay in the autocorrelations of squared high-frequency returns to the fact that market volatility reflects the aggregate impact of heterogeneous information arrivals. Under their model, market volatility has long memory. Moreover, their model implies that

Table 3: Minimum distance long memory estimate based on classical (ac) and robust (rac) autocorrelations of the absolute and squared TML filtered high-frequency EUR/USD, GBP/USD and YEN/USD returns.

	Absolute returns						Squared returns					
	EUR/USD		GBP/USD		YEN/USD		EUR/USD		GBP/USD		YEN/USD	
	ac	rac	ac	rac	ac	rac	ac	rac	ac	rac	ac	rac
5-min	0.28	0.29	0.27	0.28	0.27	0.28	0.22	0.27	0.15	0.27	0.13	0.26
10-min	0.27	0.28	0.26	0.27	0.25	0.27	0.21	0.26	0.14	0.26	0.12	0.25
15-min	0.26	0.27	0.26	0.27	0.25	0.26	0.20	0.24	0.14	0.25	0.11	0.23
20-min	0.25	0.26	0.25	0.26	0.24	0.26	0.20	0.24	0.14	0.25	0.14	0.23
30-min	0.23	0.24	0.24	0.25	0.22	0.24	0.16	0.21	0.15	0.23	0.10	0.21
1-hr	0.22	0.23	0.23	0.24	0.20	0.22	0.12	0.18	0.12	0.22	0.10	0.18

the long memory parameter d should be the same for squared and absolute high-frequency returns, and invariant to the frequency at which the returns are computed. We verify these predictions on our data in Table 3, where we report the Minimum Distance (MD) long memory parameter estimate proposed by Tieslau et al. (1996) for the k times 5-minute absolute and squared returns, with $k = 1, 2, 3, 4, 6$ and 12 (that is 5, 10, 15, 20, 30 and 60-minute returns). This estimator is defined as the value of d that minimizes the sum of squared differences between the empirical and theoretical autocorrelations under an ARFIMA(0,d,0) process:

$$\hat{d} = \mathrm{argmin}_d \sum_{l=1}^{L} (\hat{\rho}_l - \rho_l(d))^2, \tag{3.7}$$

where $\rho_l(d) = \prod_{i=1}^{l} \frac{d+i-1}{i-d}$. Through a simulation study we verified that if this estimator is based on the sample autocorrelations, jumps induce a large downward bias in the long memory estimate. If the robust autocorrelations in (3.6) are used, the MD estimate remains accurate in the presence of jumps.

We see in Table 3 that the MD long memory estimate based on classical autocorrelations of squared returns is very different from the one using absolute returns. For the GBP/USD and Yen/USD data, the long memory estimate based on the classical autocorrelation of the absolute return series is, for all frequencies, approximately twice the corresponding estimate based on squared returns. If the robust autocorrelations are used, we obtain estimates of d confirming the first prediction of Andersen and Bollerslev (1997a), namely that their value is similar for absolute and squared returns. It seems however that both under the classical and robust approach, the long memory decreases if returns are computed over lower frequencies.

4 Conclusion

This paper develops jump robust techniques for the estimation of periodicity patterns in volatility and applies them to jump detection. We propose the weighted standard deviation and truncated maximum likelihood periodicity estimators as an alternative for Taylor and Xu (1997)'s non-parametric and Andersen and Bollerslev (1997b)'s parametric periodicity estimators, respectively. The new estimators are robust to price jumps. For the estimation of sufficiently smooth periodicity patterns, we recommend the truncated maximum likelihood estimator based on the flexible Fourier form, because of its high efficiency both in the absence and presence of jumps.

We show that the robust periodicity estimates can be used to increase the accuracy of intraday jump detection methods. Our simulation study indicates that filtering matters especially for the size of the test: the original test overdetects jumps at times of periodically low intraday volatility and underdetect jumps at times of periodically high intraday volatility. Filtering is also important to increase the power of the test to detect small jumps at times of periodically low volatility, such as on Sunday evening and at the Tokyo lunch time. Using the filtered jump test statistics, we detect significantly less jumps in the 5-minute EUR/USD, GBP/USD and YEN/USD returns for the intraday intervals during which macroeconomic news are regularly released.

A Model for the periodicity factor

The regression model is similar to the one proposed by Andersen and Bollerslev (1997b). It specifies the log of the intraweek periodicity factor as a function of the Time of the Day interval $\text{ToD}_i = 1, \ldots, 288$ and Day of the Week $\text{DoW}_i = 1, \ldots, 5$ corresponding to the time point $t_i = i\Delta$:

$$\begin{aligned}
\log f_i = {} & \theta_1 \frac{\text{ToD}_i}{M_1} + \theta_2 \frac{\text{ToD}_i^2}{M_2} + \sum_{j=1}^{6} \theta_{3+j} \cos(\text{ToD}_i \frac{2\pi j}{M}) + \sum_{j=1}^{4} \theta_{9+j} \sin(\text{ToD}_i \frac{2\pi j}{M}) \\
& + (\theta_{14} + \theta_{15}\text{ToD}_i) I(\text{ToD}_i \leq 36) I(\text{DoW}_i = 1) \\
& + \sum_{d=1}^{5} \sum_{b=1}^{3} \theta_{16+3(d-1)+b} P_b(\text{ToD}_i; j) I(\text{ToD}_i \geq j_1) I(\text{DoW}_i = d) \\
& + \sum_{d=2}^{5} \sum_{a=2}^{3} \sum_{b=1}^{3} \theta_{31+6(d-2)+3(a-2)+b} P_b(\text{ToD}_i; j_a) I(\text{ToD}_i \geq j_a) I(\text{DoW}_i = d),
\end{aligned}$$

where $M_1 = M^{-1} \sum_{i=1}^{M} i = (M+1)/2$ and $M_2 = M^{-1} \sum_{i=1}^{M} i^2 = (2M^2 + 3M + 1)/6$ are normalizing constants and $P_b(t; j_a) = [1 - ((t - j_a)/M)^b](t - j_a)^{3-b}$ is a bth order Almond polynomials of t centered at j_a. We consider 3 centering points: j_1, j_2 and j_3, corresponding to the 2:30-2:35, 8:30-8:35 and 10:00-10:05 time intervals, respectively. The 2:30-2:35 polynomial is needed to accommodate for the increase in activity due to the opening of the European markets. The 8:30-8:35 and 10:00-10:05 polynomials are included to accommodate the increase in the periodic component of volatility due to the numerous releases of macroeconomic news in these intervals. The parameter vector θ has 54 components.

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