Accounting for conditional leptokurtosis and closing days effects in FIGARCH models of daily exchange rates

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Abstract

In this paper, we estimate FIGARCH models introduced by Baillie *et al.* (1996) for the four major daily exchange rates against the USD (DEM, FRF, YEN and the GBP). We extend the former contributions by accounting for the observed kurtosis through a Student-t based maximum likelihood estimation and by including variables capturing the effect of closing days. Our estimations suggest that the introduction of these features improves the goodness of fit properties of the model on the one hand, and may lead to different interest parameters estimates on the other hand. In particular, it is shown that in the case of the DEM, volatility shocks may display much less persistence than documented by previous studies. Finally, it is shown that an ARFIMA-FIGARCH framework turns out to be relevant for all the currencies (except the GBP), without inducing any significant changes in the inference of the stochastic volatility process.

Keywords : Exchange rate dynamics, Fractional Integrated ARCH, Conditional Kurtosis, Closing days effects, ARFIMA process.

JEL Classification : C22, F31, G15.

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1 Introduction

Since the past decade, the statistical analysis of financial time series has focused on the conditional second moment. Like most financial assets, exchange rate returns exhibit temporal bursts of volatility. Accordingly, exchange rate modeling has made extensive use of models that consider a stochastic form of volatility. Since the ARCH model proposed by Engle (1982), Bollerslev (1986) has introduced the GARCH framework from which the IGARCH model arises as a special case (Engle and Bollerslev, 1986).

While appealing, the infinite persistence implied by the IGARCH model appears too restrictive on the one hand, and seems contrary to empirical evidence on the other hand. In order to match more closely the observations, Baillie *et al.* (1996a) have recently introduced the Fractionally Integrated ARCH (FIGARCH hereafter) processes which provide an additional flexibility aiming at capturing long-run dependence in the conditional variance.

The FIGARCH framework has already proved to be very interesting for exchange rate modeling. Baillie *et al.* (1996a) show its relevance for the DEM-dollar case over the period 1979-1992. Quite recently, Tse (1998) applies this model to the YEN-dollar exchange rate. The results support the modeling of fractionally integrated conditional heteroskedasticity and show that it compares very well to other recent models like APARCH (Asymmetric Power ARCH) and FIAPARCH (Fractional Integrated Asymmetric Power ARCH) ones.

In this paper, we analyze the behavior of the conditional variance of the four major daily exchange rates in dollar terms (GBP, DEM, FRF and YEN) over the period 1980-1996 using the FIGARCH framework. In this respect, we extend the former empirical contributions in two directions and compare the obtained estimates. First, we do not assume conditional normality and, following Baillie and DeGennaro (1990) among others, we consider a Student-t-density aimed at accounting for the fat-tailed properties of the data. We document on these features and carry out heteroskedasticity-robust tests reported by Pagan (1996). All exchange rates are found to exhibit excess kurtosis. Second, we introduce daily effects, both in the conditional mean and in the conditional variance. Indeed, some of these effects turn out to be significant at conventional significance levels. It is therefore useful to analyze whether their introduction leads to different parameter estimates. Finally, in order to assess the robustness of the results, we allow for long run persistence in the conditional mean of the exchange rates: this is done through the estimation of a new model, the ARFIMA-FIGARCH model.

The paper is organized as follows. Section 2 provides some reliable summary statistics of the data. Section 3 presents the FIGARCH model and the preceding empirical results with respect to exchange rate modeling. Section 4 provides our empirical investigation and discusses the estimates of the extended model. Section 5 is devoted to the results of the ARFIMA-FIGARCH model. Section 6 offers some concluding remarks.

2 Statistical properties of daily exchange rates

The data consist of daily observations of four currencies in terms of the US dollar - the British pound (GBP), the Japanese yen (YEN), the French franc (FRF) and the Deutsche mark (DEM) - from 1980 to 1996 (T=4220 observations for the YEN and 4312 for the other currencies). The data are provided by the International Bank for Settlements. Starting from the usual stylized facts in terms of the nonstationarity of the raw spot exchange rate series y_t (Meese and Singleton (1982), Baillie and Bollerslev (1989)), we shall concentrate on the modelling of the daily nominal percentage returns, i.e. $r_t = 100 \ln(y_t/y_{t-1})$ for $t = 1, 2, ..., T$, which have to be integrated of order zero.

The empirical literature on foreign exchange rates typically finds no linear temporal dependence (no or little autocorrelation) but a second-order non-linear temporal dependence, i.e. strong heteroskedasticity (i.e. small and large changes are clustered over time). Moreover, the distribution of exchange rate returns is generally bell-shaped, asymmetric and exhibit fat tails (Diebold and Nerlove, 1989; Bollerslev, 1987; Baillie and Bollerslev, 1989; Hsieh, 1988). Table 1 gives the first 10 autocorrelation coefficients and the (heteroskedasticity consistent *a la* Diebold[1]) Box-Pierce statistic for different lags, whereas the autocorrelation coefficients and the Box-Pierce statistic of the squared data are reported in the Table 2.

[INSERT TABLE 1]

In Table 1, serial correlation is detected for the FRF and the DEM with the first autocorrelation coefficients significant at the 1% level.[2] An ARMA(1, 0) model on these two returns series proves to be adequate to account for serial correlation. The squared data in Table 2, which have been filtered by an ARMA(1, 0) model for the FRF and the DEM exhibit strong autocorrelation. This may be indicative of conditional heteroskedasticity. A second test of heteroskedasticity, the Levene modified test of Brown and Forsythe (1974), which presents the advantage of being robust to departures from normality, is also provided in Table 2. This test suggests strong heteroskedasticity in the data, too.

[INSERT TABLE 2]

It is also interesting to assess the departure from the normality hypothesis. Therefore, we focus upon the higher order moments of returns, i.e. the skewness and excess kurtosis parameters, denoted respectively b_3 and b_4. In order to take into account the suspected temporal dependence in the squared data, we make the t-statistics of these tests robust to this dependence, along the lines proposed by Pagan (1996). Finally, a test for general non-linear dependence is implemented, too: we use the B.D.S. test (Brock *et al.*, 1987). The results of these tests are reported in Table 3.

[INSERT TABLE 3]

The kurtosis coefficients are all substantially larger than the value corresponding to the standard distribution. This suggests that the empirical distribution of the returns deviates from the normal one in that it exhibits heavy tails (the observations are concentrated around the mean and there are many outliers compared to the normal distribution). In lines with the findings of Pagan (1996), no asymmetry of the distributions is detected when the t-statistics are made robust to the dependence. These results contrast with the findings of previous works which generally conclude in favor of asymmetry (Baillie and Bollerslev, 1989; Hsieh, 1989; Palm and Vlaar, 1997). Finally, the B.D.S. statistics, combined with the strong autocorrelation of squared data detected in Table 2, indicate substantial non-linear dependence in the data. We implement the Hsieh test (1989) in order to distinguish between two types of non-linear dependence, i.e. additive and multiplicative dependence. The results of this test (not reported here) conclude to multiplicative dependence, which justifies the standard autoregressive conditional heteroskedasticity (ARCH/GARCH and related) formulations.

3 The FIGARCH framework

The FIGARCH $(1, d, 1)$ model introduced by Baillie *et al.* (1996a) is given by the two following equations:

$$r_t = \mu + \rho r_{t-1} + \epsilon_t, \qquad \epsilon_t \mid \Omega_t \sim D(0, \sigma_t^2) \tag{1}$$

$$\sigma_t^2 = \omega + \left\{ 1 - [1 - \beta_1 L]^{-1} (1 - \phi_1 L) [1 - L]^d \right\} \epsilon_t^2 \tag{2}$$

where an AR1 process is allowed for r_t, μ is the mean of the process and Ω_t is the information set at time t. ρ, ω, β_1, ϕ_1, d are parameters to be estimated with d being the fractional integration parameter and finally, L is the lag operator.[3] Of course, in the conditional variance, the model may be generalized to higher AR and MA orders. For a FIGARCH$(1, d, 1)$, sufficient conditions for the conditional variance to be strictly positive are given in Baillie *et al.* (1996a).[4] For higher AR and MA orders, these conditions are cumbersome to derive, which obviously hampers the generalisation of the FIGARCH specification to higher orders. Therefore, in this paper, we will stick to the FIGARCH$(1, d, 1)$ specification. Interestingly, the FIGARCH$(1, d, 1)$ model nests the GARCH(1,1) model (Bollerslev 1986) for $d = 0$ and the IGARCH model (Engle and Bollerslev, 1986) for $d = 1$. As advocated by Baillie *et al.* (1996a), the IGARCH process may be seen as too restrictive as it implies infinite persistence of a volatility shock. Such a dynamics is contrary to the observed behavior of agents and does not match the persistence observed after important events (see Baillie *et al.*, 1996a; Bollerslev and Engle, 1993). By contrast, for $0 < d < 1$, the FIGARCH model implies a long-memory behavior and a slow decay of the impact of a volatility shock. As reported below, the early findings in favor of an integrated variance have been invalidated on exchange rate data in two recent studies.

The FIGARCH framework has indeed been found useful in exchange rate modeling. On the one hand, Baillie *et al.* (1996a) estimate a FIGARCH model on the daily DEM-US Dollar exchange rate from March 1979 through December 1992. On the other hand, Tse (1998) applies this framework to the YEN-US Dollar exchange rate from January 1978 through June 1994. Both papers assume conditional normality and retain a FIGARCH$(1, d, 0)$ model. In the Gaussian case for D, the log-likelihood of the model takes the following form :

$$Ln(L_{norm}) = \sum_{t=1}^{T} \left[-0.5 \ln \left(2\pi\sigma_t^2 \right) + (\epsilon_t^2/\sigma_t^2) \right] \quad (3)$$

In order to make the comparisons with our results easier, Table 4 provides the findings for these preferred models.

[INSERT TABLE 4]

The estimates of the fractional integration parameter d turn out to be quite different across the two exchange rates. While the FIGARCH model estimated by Baillie *et al.* (1996a) may be mistaken as an IGARCH-type model, Tse (1998) concludes that there is no substantial difference between the estimated FIGARCH model and a stable one. In turn, this raises the question whether such a heterogeneity exists between the major exchange rates dynamics. It is one of the purposes of this paper to compare over an extended period (1980-1996) the dynamics of the four major exchange rates.

On the methodological side, we extend the two above mentioned contributions in two directions. First, both papers assume conditional normality. This normality assumption is to a certain extent justified by the fact that the quasi maximum likelihood estimates (QMLE) are found to behave quite well, even when non normal errors are considered (see Baillie *et al.*, 1996a on this point). Nevertheless, it may be expected that excess kurtosis displayed by the residuals of conditional heteroskedasticity models will be reduced when a more appropriate distribution is used. As reported by Palm (1996), Pagan (1996) and Bollerslev *et al.* (1992), the use of a Student-t distribution is widespread in the literature. In particular, Bollerslev (1987), Hsieh (1989), Baillie and Bollerslev (1989) and Palm and Vlaar (1997) among others show that this distribution performs better in order to capture the higher observed kurtosis. In the case of a Student-t distribution, the log-likelihood to be maximized becomes:

$$\begin{aligned} Ln(L_{St-t}) &= T\left[\ln\Gamma\left\{(\nu+1)/2\right\} - \ln\Gamma\left\{\nu/2\right\} - 1/2\ln\pi(\nu-2)\right] - \\ & 1/2\sum_{t=1}^{T}\left\{\ln(\sigma_t^2) + (\nu+1)\left[\ln\left(1+\frac{\epsilon_t^2}{\sigma_t^2(\nu-2)}\right)\right]\right\} \end{aligned} \quad (4)$$

where, $\Gamma(\cdot)$ is the gamma function. Since the Student-t nests the Normal distribution, statistical tests like likelihood ratio tests (LRT) can be used. Compared to the normal distribution, the Student-t indeed implies the estimation of the additional parameter ν standing for the number of degrees of freedom.

The second extension considers the inclusion of additional variables aiming at refining the modeling of the stochastic volatility and the conditional mean of exchange rate returns. For financial daily data, it may be expected that trading recovery or the prospect of a market closure may influence market activity and thus explains some part of the dynamics. Notice that this point has been considered by some authors (Baillie and Bollerslev, 1989; Hsieh 1989; Palm and Vlaar, 1997). In this paper, we consider two analogous effects (hereafter referred as "closing day effects"). We introduce two variables, $x_{1,t}$ and $x_{2,t}$, standing for the number of vacation days respectively before and after the day under consideration. Notice that the introduction of middle-week days dummies did not seem to capture any effect.[5] In order to avoid the imposition of any a priori restriction, the variables $x_{1,t}$ and $x_{2,t}$ are introduced both in the mean and the variance equations.[6] Model specification becomes:

$$\epsilon_t = \sigma_t e_t + \alpha_1 x_{1,t} + \alpha_2 x_{2,t} \quad (5)$$

$$\sigma_t^2 = (\omega + \alpha_3 x_{1,t} + \alpha_4 x_{2,t}) + \left\{1 - \left[1-\beta_1 L\right]^{-1}(1-\phi_1 L)\left[1-L\right]^d\right\}\epsilon_t^2 \quad (6)$$

Once more LRT can be used to statistically assess the relevance of the inclusion of these daily effects.

4 Empirical Results

The FIGARCH model extended with closing days effect and estimated on the basis of a Student-t distribution seems to suit well the exchange rate data regardless of the currency under investigation. Nevertheless, our aim is not only to provide a relevant framework but also to assess to which extent a different specification and/or a different underlying distribution may lead to different parameters estimates. Therefore, for each currency, we provide the results of various competing models. Basically, as the models are nested, we can carry out LRT such as to retain a preffered model. We then draw the implications of these findings in terms of model specification and estimation.

Our model comparison rests on 8 different specifications. Indeed, we make a distinction regarding three distinct model features. First, we estimate the models relying either on a normal distribution or a Student-t with ν, the number of degrees of freedom being an additional parameter to be estimated. Second, we introduce the closing days effects, both in the conditional mean and in the conditional variance. Finally, we make a clear distinction between the FIGARCH$(1, d, 1)$ framework and the FIGARCH$(1, d, 0)$. While the former one nests the latter, it is still interesting to measure to which extent a misspecified framework (through omitting to introduce the significant ϕ_1 parameter) may lead to different d estimates. This question is of primarily importance as the preferred models of Baillie *et al.* (1996a) and Tse (1998) are FIGARCH$(1, d, 0)$ models.

Tables 5 through 8 report for each currency the eight estimated models. The statistics reported in section 2 on the raw data - the skewness and excess kurtosis parameters b_3 and b_4, the Box-Pierce statistics[7] $Q(20)$ and $Q^2(20)$ and the t-stat of the BDS statistics, denoted $BDS(a)$ for a (the embedding dimension) set equal to 6 - are also computed on the standardized residuals of estimated models for diagnostic purposes. The lasts two rows provide the p-values of the LRT aiming at comparing the nested models. The first one gives the results of LRT relative to the choice of the underlying distribution of the errors.[8] The second row reports the p-values of the LRT aiming at assessing the relevance of the inclusion of the closing days effects.[9]

[INSERT TABLES 5, 6, 7 and 8]

From these four tables, five main comments are in order.

1. The relevance of the Student-t distribution is confirmed. The t-value for ν, which leads to similar conclusions for $1/\nu$, is highly significant for all currencies. LRT clearly indicate that the Student-t outperforms the Normal distribution. Regardless model specification, the value of b_4 is quite close to zero, even though this is less obvious for the YEN. Thus, the Student-t distribution seems to do a good job in accounting for kurtosis, which is not the case for the normal distribution. At some lags, the squared residuals from the normal specification exhibit serial correlation whereas it is not the case for the Student-t based ones. Examining the BDS test statistics, the null of i.i.d. residuals is never rejected for any model estimated with the Student-t assumption, which is not the case for the Normal distribution. This is especially obvious for the FRF and the YEN for which the i.i.d. hypothesis is often rejected for models estimated with the normal density but not with the Student-t one. While the goodness-of-fit of the Student-t based model seems as a whole superior, the values of the parameter estimates are not found to depend significantly on the underlying distribution (except for the YEN). This result is consistent with the findings of Baillie *et al.* (1996a) showing that the QML estimators behave very well, even in the presence of non normal errors.

2. The inclusion of closing days effects seems meaningful, regardless model specification. LRT (see last rows) clearly supports their inclusion for all currencies. In particular, it is found that trading recovery is associated with increasing volatility. This result is consistent with the findings of Hsieh (1988), Baillie and Bollerslev (1989) and Palm and Vlaar (1997) who detect significant daily effects in the variance. More precisely, the fact that α_3 turns out to be significantly positive is in line with Hsieh (1988) who found a higher conditional variance on Mondays but not on Fridays. This effect may be due to important reallocations in portofolios implied by the processing of information taking place during the vacation days. The introduction of the closing days effects in the mean seems in general less justified, although significant impacts are found for the DEM, the FRF and the YEN.

   The introduction of these effects improves model's adequacy. This is obvious from the diagnostic tests reported in Tables 5-8. In particular, it is found that the excess kurtosis is often reduced for all currencies and across all models. The usual explanation for this phenomenon is that volatility reflects the volume of trading and the flow of information to the market as hypothesized for example by French and Roll (1986) and Caporale *et al.* (1998). Also, the inclusion of the closing days effects leads to an important decrease of $Q^2(20)$, which can be used as a mispecification test (see Bollerslev and Mikkelsen, 1996 on this point). In turn, this changes the inference on the pattern of volatility shocks: for the Student-t distribution (which is the preferred one as outlined above), the volatility shocks display less persistence when accounting for closing days effects. This may also explain why Baillie *et al.* (1996a) found relatively high values for d, although another cause will be emphasized later.

3. Some of our findings are broadly consistent with the previous ones of the literature. As in the previous studies, the d parameter is found significantly different from 1 at the 5% nominal level, which clearly rejects the IGARCH process for the daily exchange rates dynamics. For the YEN, the same model , i.e. the FIGARCH$(1, d, 0)$ yields a d estimate very close from the one found by Tse (1998).

4. From LRT (not reported explicitely in the tables) and statistical tests on ϕ_1,the retained specification turns out to be the FIGARCH$(1, d, 1)$ model with closing days effects and with the assumption of Student-t errors. The choice of this model turns out to be important for the inference of the d parameter. For the DEM, the d parameter is equal to about 0.43, much in line with the results for the other currencies. This contrasts with the findings of Baillie *et al.* (1996a) who conclude in favor of a very high long run dependence in the conditional variance. Our results imply that the volatility process is much closer to a simple GARCH process than to an IGARCH one. Furthermore, our results conclude in favor of a similar pattern of volatility persistence across the major currencies.

   Four main causes of the diverging results with Baillie *et al.* (1996a) can be put forward. The first one is related to the inclusion of daily effects as underlined above. The second one may be due to the selection of a wrong model (the FIGARCH$(1, d, 0)$ while the right one is a FIGARCH$(1, d, 1)$). This selection can be it-self influenced by the choice of the underlying distribution and the inclusion of the closing days effects, as illustrated by the case of the FRF.[10] A third origin may lie in the choice of the starting values in the ML estimation. In order to investigate further this point, we compute the normed log-likelihood profiles for the FIGARCH$(1, d, 1)$ and $(1, d, 0)$ models of the DEM exchange rate. These are reported in Figures 1 to 2. It is shown that, for the FIGARCH$(1, d, 0)$, depending on the initial values for the parameters vector, one may easily converge to a value of d implying a strong long run dependence in the volatility process. Interestingly, this local maximum (lying around $d = 0.87$) turns out to be rather close to the solution found by Baillie *et al.* (1996a) (see also Table 4). Even importantly, as suggested by Figure 1, the case for multiple maxima does not hold for the FIGARCH$(1, d, 1)$ model. Finally, one cannot exclude that the choice of the sample period plays an important role. Combined with our results, the Baillie et al. (1996) findings could suggest that the d parameter has shifted during the investigated sample periods.

[INSERT FIGURES 1 and 2]

5. To sum up, the inclusion of the closing days effects seems to affect the estimation of d by two channels, at least for our particular sample period. The first one is a direct one, similar to the one arising in accounting for seasonality in time series models. The second one is related to model selection and is thus indirect: the inclusion of daily effects leads to the selection of the FIGARCH$(1, d, 1)$ model (instead of a FIGARCH$(1, d, 0)$) for which there is no case of local maxima.

5 Allowing for a fractional unit root in the mean

Up to now, the analysis has mainly focused on the conditional volatility of the model, whereas less attention has been paid to the modeling of the conditional mean. So far, it has been assumed that exchange rates levels exhibit a (deterministic) unit root so that first differencing takes place in the mean equation. Nevertheless, several studies like Cheung (1993a) have pointed out the possibility that exchange rates returns measured on a high frequency basis follow a fractionally integrated process. Such a dynamics is best captured through a fractionally integrated ARMA process (so called ARFIMA process) initially developed by Granger (1980), Granger and Joyeux (1980) among others. Granger and Terasvirtä (1993) suggest that mispecification in the conditional mean may affect the estimation of the conditional variance parameters. Through Monte Carlo simulations, Teyssière (1997) illustrates that ignoring the long memory in the conditional mean of a double long memory process (i.e. long run dependence in both the mean and the variance) can lead to serious biases in the estimation of the conditional volatility process. Consequently, in order to assess the robustness of our FIGARCH models estimation, we introduce the possibility of a fractional root in the mean.

The estimation of ARFIMA processes with time-dependent heteroskedasticity is fairly new. Baillie *et al.* (1996b) estimate an ARFIMA(n, ζ, s)-GARCH(p, q) process for the post-war inflation rates of several industrial countries. The estimation of the ARFIMA(n, ζ, s)-FIGARCH(p, d, q) is fairly new in the literature (see nevertheless Teyssière 1997 and 1998). Allowing for such a process, equation (2) remains unchanged but equation (1) becomes:

$$\Psi(L)(1-L)^{\zeta}\left[r_t - (m + \alpha_1 x_{1,t} + \alpha_2 x_{2,t})\right] = \Theta(L)\,\epsilon_t \tag{7}$$

where m is the mean of the process, $\zeta = 0$ corresponds to a stationary process for the exchange rate returns (r_t). $\Psi(L) = 1 - \psi_1 L - ... - \psi_n L^n$ and $\Theta(L) = 1 + \theta_1 L + ... + \theta_s L^s$ are the usual AR and MA polynomials of respective order n and s for which all roots lie outside the unit circle. With $\Psi(L) = 1 - \rho L$, $\zeta = 0$ and $\Theta(L) = 1$, process (7) restricts to the process considered in section 4, i.e. a first differenced stationary process with an AR1 term. If $\zeta \in (0, 1/2)$, the process is stationary and has a long memory or is said to be persistent. If $\zeta \in (-1/2, 0)$,the process has a short memory or is said to be antipersistent.[11]

Following Teyssière (1997), the model is estimated by ML as well. The ARFIMA-FIGARCH model requires nevertheless the choice of a model selection procedure in order to determine the orders n and s.[12] In this respect, we rely on an information criterion (Schwarz Bayesian Criterion, see Schwarz, 1978) which leads to a parsimonious model.[13] This choice is made for two reasons. First, this is made for comparison purposes with the previous FIGARCH models in which the conditional mean was allowed to display an AR1 term. Second, the inclusion of higher orders turns out to induce relatively high estimated values for the AR and the MA coefficients. In turn, as Cheung (1993b) shows through Monte Carlo results, the behavior of ML tests for fractional roots is highly sensitive to the occurence of large AR and MA components. Note also that the other information criteria may lead to the choice of different orders but with the exception of the GBP, the inference about ζ and the conditional variance remains similar. Since our analysis primarily aims at assessing the robustness of the FIGARCH results and in order to save space, we only report the retained ARFIMA-FIGARCH$(1, d, 1)$ models. For all, the estimation procedure assumes ϵ_t to follow a Student-t distribution and closing days effects are introduced.

Like for the FIGARCH estimations, one can assess the relevance of the ARFIMA-FIGARCH framework on the basis of LRT. For meaningful comparisons however, the constrained model must display the same ARMA structure. Therefore, in each case, we reestimate an ARMA-FIGARCH framework whose ARMA terms in the conditional mean are the same as the one selected for the ARFIMA-FIGARCH model. Table 9 reports the results for the four currencies. Like for the FIGARCH results, the last row provides the p-values of the LRT mentionned above.

[INSERT TABLE 9]

Looking at Table 9, one may draw two main conclusions. First, for three currencies (DEM, FRF and YEN), the results seem to (weakly) support the hypothesis of a fractional root in the mean since ζ turn out to be significant the 5% level (but not at the 1% level). The LRT lead to the same conclusions. In other terms, the ARFIMA-FIGARCH turns out to capture more or less the dynamics of daily exchange rates. For the GBP, the null of a fractional root is rejected, which is in line with the findings of Cheung (1993a)[14] and the results of the Geweke-Porter-Hudak test (not reported here to save place). For the other exchange rates returns, the inferred fractional root is quite low, but exchange rate returns are neverthelss found to be slightly persistent. Second, the estimated dynamics in the conditional variance seems unaffected by the inclusion of a fractional root. To a certain extent, this is unsurprizing since the value of ζ is quite low.[15] Notice nevertheless that similar results are obtained by Baillie *et al.* (1996b) for higher values of ζ, but within a GARCH framework. As a whole, the ARFIMA-FIGARCH analysis emphasizes the robustness of the results of the FIGARCH analysis.

6 Conclusion

In this paper, we apply the FIGARCH framework to the four major daily exchange rates in dollar terms on the period 1980-1996. We extend the former contributions by relying on the Student-t distribution in the ML estimation and by introducing closing days effects in the modeling of the conditional mean and variance. The introduction of these features proves to be relevant: exchange rate volatility is found to be amplified after market closure; the observed excess kurtosis seems to be reduced by dropping the normality assumption.

Importantly, the inclusion of these features seems to affect the estimated value of the fractional parameter in the variance. This is especially the case for closing days effects. One the one hand, accounting for vacation days leads to lower estimates of this parameter. On the other hand, failing to control for these effects may lead to the selection of a wrong model, whose ML estimation faces problems of local maxima. As a whole, our results suggest that the persistence of volatility shocks is much less important than reported by previous studies like Baillie *et al.* (1996) and that the four currencies exhibit in this respect similar patterns. Allowing for a fractional root in the conditional mean turns out to be to some extent relevant but does not lead to other parameter estimates relative to the volatility sides.

We think that this analysis deserves several refinements. While adequate to reduce some excess kurtosis, the use of a Student-t distribution still leaves some excess skewness in the residuals, which may be due to some jumps triggered by the management of floating exchange regimes and the numerous agreements between the major central banks during that particular period. Along the lines of Vlaar and Palm (1993) and Palm and Vlaar (1997), it would be interesting to check whether normal mixture distributions can better match the observed dynamics.

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TABLES

Table 1 : Autocorrelation of Raw Data

Autocorrelation coefficients	GBP	YEN	FRF	DEM
$\rho_x(1)$	-0.005	-0.013	-0.054	-0.051
	(-0.285)	(-0.685)	(-2.688)	(-2.820)
$\rho_x(2)$	0.023	0.003	0.008	0.011
	(1.277)	(-0.153)	(0.516)	(0.657)
$\rho_x(3)$	-0.002	0.016	0.043	0.027
	(0.122)	(0.281)	(2.147)	(1.442)
$\rho_x(4)$	-0.002	0.027	-0.022	-0.008
	(-0.104)	(1.355)	(-1.144)	(-0.429)
$\rho_x(5)$	0.027	-0.014	0.023	0.016
	(1.441)	(-0.764)	(1.306)	(0.951)
$\rho_x(6)$	0.016	-0.004	0.005	0.007
	(0.799)	(-0.242)	(0.320)	(0.379)
$\rho_x(7)$	-0.034	0.010	-0.003	-0.006
	(-1.852)	(0.608)	(-0.212)	(-0.334)
$\rho_x(8)$	-0.006	0.016	-0.007	0.001
	(-0.353)	(0.915)	(-0.428)	(0.041)
$\rho_x(9)$	0.040	0.025	0.042	0.036
	(2.210)	(1.359)	(2.419)	(2.107)
$\rho_x(10)$	-0.003	0.037	-0.018	-0.014
	(-0.140)	(2.146)	(-1.021)	(-0.779)
Adjusted Box-Pierce				
$Q_x(20)$	24.932	24.701	38.555	36.593
	[0.203]	[0.213]	[0.007]	[0.013]

t-statistics in parentheses and marginal significance level in brackets.

Table 2 : Autocorrelations of Squared Data (linearly filtered by an ARMA(1,0) model for the franc and the deutschmark)

Autocorrelations	GBP	YEN	FRF	DEM
$\rho_{xx}(1)$	0.128	0.080	0.072	0.078
	(3.606)	(3.691)	(4.672)	(5.069)
$\rho_{xx}(2)$	0.080	0.104	0.031	0.037
	(3.502)	(2.628)	(2.078)	(2.467)
$\rho_{xx}(3)$	0.127	0.063	0.095	0.107
	(3.594)	(2.965)	(6.174)	(6.853)
$\rho_{xx}(4)$	0.117	0.118	0.084	0.104
	(3.020)	(2.675)	(5.466)	(6.686)
$\rho_{xx}(5)$	0.080	0.073	0.054	0.051
	(3.618)	(3.492)	(3.557)	(3.331)
$\rho_{xx}(6)$	0.083	0.047	0.054	0.022
	(3.684)	(2.674)	(3.545)	(5.292)
$\rho_{xx}(7)$	0.081	0.039	0.063	0.069
	(2.975)	(2.056)	(4.087)	(4.449)
$\rho_{xx}(8)$	0.072	0.060	0.050	0.062
	(3.401)	(2.888)	(3.269)	(4.009)
$\rho_{xx}(9)$	0.072	0.080	0.043	0.047
	(3.401)	(2.606)	(2.820)	(3.088)
$\rho_{xx}(10)$	0.108	0.050	0.047	0.074
	(2.191)	(2.660)	(3.108)	(4.8114)
Box-Pierce				
$Q_{xx}(20)$	351.72	289.71	215.83	330.82
	[0.000]	[0.000]	[0.000]	[0.000]
Modified Levene test				
F(203,4108)	1.38	1.81	1.40	1.34
	[0.000]	[0.000]	[0.000]	[0.000]

t-statistics in parentheses and marginal significance level in brackets.
The modified Levene test follows an F(203,4108) distribution.

Table 3 : Testing departure from normality for returns data (filtered by an ARMA(1,0) model for the FRF and the DEM)

Statistics	GBP	YEN	FRF	DEM
b_3	-0.04	-0.43	-0.06	-0.28
	(-0.17)	(-1.83)	(-0.25)	(-1.15)
b_4	4.34	3.79	5.76	3.70
	(2.79)	(3.57)	(2.76)	(2.75)
B.D.S(6)	18.15	15.54	14.69	12.06

note: b_3 and b_4 are skewness and excess kurtosis parameters. This tests should be referred to a N(0,1) random variable. Figures in parentheses are the robust t-statistics. The B.D.S. test was constructed by setting the length of the correlation integral to one time the standard deviation of the series and the dimension of this integral *m* to six.

Table 4 : Former FIGARCH models for the dem and the yen

Parameters	dem (Baillie et al 1996)	yen (Tse 1998)
μ	-0.004	-0.011
ω	0.017 *	0.123 *
β_1	0.762 *	0.121 *
d	0.823 *	0.191 *

Sources : Baillie *et al.* (1996a), Tse (1998);
* indicates significance at 1% level

Table 5 : FIGARCH models for the GBP-USD exchange rate

(p,d,q)	*(1,d,0)*				*(1,d,1)*			
Distribution	Normal	Student	Normal	Student	Normal	Student	Normal	Student
	No Days Effect		Days Effects		No Days Effect		Days Effects	
μ	-0.002	-0.005	-0.001	-0.008	-0.002	-0.0046	-0.0010	-0.0074
	(-0.255)	(-0.624)	(-0.095)	(-0.786)	(-0.210)	(-0.581)	(-0.099)	(-0.769)
ρ	-0.001	-0.023	-0.013	-0.027	-0.015	-0.0269	-0.0171	-0.0291
	(-0.540)	(-1.484)	(-0.785)	(-1.717)	(-0.881)	(-1.723)	(-1.028)	(-1.903)
α_1	-	-	-0.013	-0.008	-	-	-0.0103	-0.0072
			(-0.986)	(-0.674)			(-0.762)	(-0.583)
α_2	-	-	0.008	0.012	-	-	0.0060	0.0105
			(0.745)	(1.185)			(0.562)	(1.068)
ω	0.038	0.032	0.005	0.001	0.0298	0.0258	-0.0048	-0.0102
	(4.039)	(3.805)	(0.584)	(0.142)	(3.117)	(2.708)	(-0.458)	(-1.116)
β_1	0.183	0.229	0.152	0.196	0.6685	0.6741	0.6069	0.6314
	(3.780)	(4.864)	(3.254)	(4.457)	(11.028)	(13.385)	(7.169)	(10.495)
ϕ_1	-	-	-	-	0.3683	0.3295	0.3628	0.3346
					(6.596)	(6.992)	(5.223)	(6.346)
d	0.312	0.330	0.289	0.304	0.4481	0.4678	0.3962	0.4227
	(9.464)	(9.999)	(8.648)	(9.724)	(7.247)	(8.884)	(5.842)	(7.568)
α_3	-	-	0.092	0.088	-	-	0.0986	0.1003
			(5.542)	(5.653)			(5.670)	(5.866)
α_4	-	-	0.010	0.009	-	-	0.0115	0.0117
			(1.271)	(1.202)			(1.300)	(1.293)
ν	-	5.572		7.007	-	6.5480	-	6.8560
		(11.914)		(10.782)		(11.692)		(10.574)
b_3	-0.194	0.078	0.099	0.122	0.050	0.057	0.077	0.084
b_4	1.481	-2.286	1.297	-0.631	1.442	-0.871	1.312	-0.723
Q(20)	48.691 ***	48.474 ***	44.554 ***	45.136 ***	46.704 ***	48.593 ***	45.938 ***	47.880 ***
$Q^2(20)$	36.617 ***	35.454 ***	33.514 ***	33.515 ***	29.237 *	32.209 ***	22.860	25.603
BDS(6)	-1.674	-1.324	-1.466	-1.150	-0.940	-0.328	-0.950	-0.281
Log Lik	-4277.1	-4184.3	-4224.1	-4147.3	-4262.0	-4172.3	-4215.9	-4139.1
Distribution		[0.000]		[0.000]		[0.000]		[0.000]
Daily effects			[0.000]	[0.000]			[0.000]	[0.000]

Note: asymptotic heteroskedasticity-consistent t-statistics are in parentheses and p-values of the LRT tests are in brackets.

Model: $r_t = \mu + \alpha_1 x_{1,t} + \alpha_2 x_{2,t} + \rho r_{t-1} + \epsilon_t$ and

$$\sigma_t^2 = (\omega + \alpha_3 x_{1,t} + \alpha_4 x_{2,t}) + \left\{1 - [1 - \beta_1 L]^{-1} (1 - \phi_1 L) [1 - L]^d \right\} \epsilon_t^2$$

Table 6 : FIGARCH models for theYEN-USD exchange rate

(p,d,q)	*(1,d,0)*				*(1,d,1)*			
Distribution	Normal	Student	Normal	Student	Normal	Student	Normal	Student
	No Days Effect		Days Effects		No Days Effect		Days Effects	
μ	-0.003	0.014	-0.021	-0.008	-0.0038	0.0137	-0.0215	-0.0090
	(-0.311)	(1.664)	(-1.858)	(-0.789)	(-0.388)	(1.638)	(-1.960)	(-0.875)
ρ	-0.015	-0.043	-0.020	-0.046	-0.0150	-0.0441	-0.0183	-0.0450
	(-0.881)	(-2.779)	(-1.189)	(-2.940)	(-0.890)	(-2.810)	(-1.097)	(-2.899)
α_1	-	-	0.026	-0.028	-	-	0.0255	0.0284
			(1.830)	(-2.211)			(1.815)	(2.269)
α_2	-	-	0.018	0.022	-	-	0.0180	0.0224
			(1.467)	(2.209)			(1.505)	(2.265)
ω	0.079	0.039	0.146	0.004	0.0824	0.0480	0.0084	-0.0007
	(2.718)	(3.748)	(0.777)	(0.466)	(2.894)	(3.378)	(0.399)	(-0.056)
β_1	0.155	0.303	0.145	0.267	0.4198	0.5751	0.4174	0.5412
	(2.208)	(3.892)	(2.329)	(4.221)	(3.679)	(8.688)	(3.859)	(7.639)
ϕ_1	-	-	-	-	0.2340	0.2188	0.2438	0.2221
					(2.573)	(3.948)	(2.532)	(3.836)
d	0.256	0.386	0.260	0.354	0.2982	0.4684	0.3000	0.4297
	(4.383)	(5.856)	(5.386)	(6.682)	(3.613)	(7.061)	(4.612)	(6.839)
α_3	-	-	0.129	0.085	-	-	0.1495	0.1131
			(3.882)	(5.157)			(4.112)	(5.891)
α_4	-	-	0.016	0.003	-	-	0.0160	0.0036
			(1.029)	(0.471)			(0.915)	(0.375)
ν	-	5.304		5.638	-	5.2735	-	5.5897
		(12.252)		(11.680)		(12.505)		(11.737)
b_3	-0.194	-0.724	-0.387	-0.481	-0.573	-0.602	-0.344	-0.358
b_4	5.514	1.379	4.212	0.378	5.574	1.829	4.294	1.326
Q(20)	48.691 ***	48.474 ***	44.554 ***	45.136 ***	46.704 ***	48.593 ***	45.938 ***	47.880 ***
$Q^2(20)$	36.617 ***	35.454 ***	33.514 ***	33.515 ***	29.237 *	32.209 ***	22.860	25.603
BDS(6)	3.025	0.862	1.733	0.691	3.343	1.814	2.093	1.743
Log Lik	-4336.5	-4126.2	-4263.6	-4086.2	-4340.7	-4128.5	-4264.6	-4088.9
Distribution		[0.000]		[0.000]		[0.000]		[0.000]
Daily effects			[0.000]	[0.000]			[0.000]	[0.000]

Note: see Table 5

Table 7 : FIGARCH models for the FRF-USD exchange rate

(p,d,q)	*(1,d,0)*				*(1,d,1)*			
Distribution	Normal	Student	Normal	Student	Normal	Student	Normal	Student
	No Days Effect		Days Effects		No Days Effect		Days Effects	
μ	0.002	0.008	-0.004	-0.001	0.0015	0.0092	-0.0044	0.0001
	(0.213)	(1.037)	(-0.325)	(-0.022)	(0.163)	(1.097)	(-0.378)	(0.009)
ρ	-0.043	-0.044	-0.048	-0.052	-0.0460	-0.0463	-0.0511	-0.0522
	(-2.634)	(-2.937)	(-2.947)	(-3.446)	(-2.721)	(-3.061)	(-3.172)	(-3.492)
α_1	-	-	-0.010	-0.008	-	-	-0.0111	-0.0081
			(-0.713)	(-0.631)			(-0.759)	(-0.604)
α_2	-	-	0.020	0.026	-	-	0.0200	0.0256
			(1.743)	(2.478)			(1.757)	(2.450)
ω	0.016	0.035	0.006	0.009	0.0543	0.0408	0.0049	0.0015
	(3.064)	(4.002)	(0.837)	(1.029)	(3.084)	(3.143)	(0.302)	(0.124)
β_1	0.789	0.328	0.255	0.243	0.6811	0.6193	0.5692	0.5566
	(13.141)	(3.212)	(2.125)	(3.717)	(8.428)	(10.448)	(4.116)	(7.599)
ϕ_1	-	-	-	-	0.1583	0.2156	0.2102	0.2332
					(2.005)	(4.422)	(2.970)	(4.741)
d	0.882	0.390	0.347	0.316	0.6343	0.4899	0.4723	0.4113
	(11.388)	(4.390)	(3.171)	(5.690)	(4.703)	(6.336)	(2.526)	(4.988)
α_3	-	-	0.105	0.094	-	-	0.1287	0.1171
			(3.328)	(5.082)			(4.665)	(5.918)
α_4	-	-	0.005	0.003	-	-	0.0029	0.0012
			(0.552)	(0.330)			(0.253)	(0.097)
ν	-	6.175		6.556	-	6.1891	-	6.4478
		(10.185)		(9.829)		(10.849)		(9.948)
b_3	-0.194	-0.107	-0.067	-0.095	-0.085	-0.071	-0.081	-0.076
b_4	2.863	0.629	2.544	0.330	3.062	0.569	2.476	0.183
Q(20)	48.691 ***	48.474 ***	44.554 ***	45.136 ***	46.704 ***	48.593 ***	45.938 ***	47.880 ***
$Q^2(20)$	36.617 ***	35.454 ***	33.514 ***	33.515 ***	29.237 *	32.209 ***	22.860	25.603
BDS(6)	-2.182	-0.969	-1.698	-0.723	-1.755	-0.240	-1.210	0.310
Log Lik	-4515.1	-4382.3	-4460.5	-4344.7	-4509.1	-4373.4	-4452.5	-4338.4
Distribution		[0.000]		[0.000]		[0.000]		[0.000]
Daily effects			[0.000]	[0.000]			[0.000]	[0.000]

Note: see Table 5

Table 8 : FIGARCH models for the DEM-USD exchange rate

(p,d,q)	*(1,d,0)*				*(1,d,1)*			
Distribution	Normal	Student	Normal	Student	Normal	Student	Normal	Student
	No Days Effect		Days Effects		No Days Effect		Days Effects	
μ	0.004	0.008	-0.004	-0.003	0.0042	0.0090	-0.0039	-0.0028
	(0.366)	(0.894)	(-0.360)	(-0.789)	(-0.388)	(1.638)	(-1.960)	(-0.875)
ρ	-0.047	-0.051	-0.056	-0.058	-0.0545	-0.0537	-0.0588	-0.0593
	(-2.654)	(-3.423)	(-3.451)	(-2.940)	(-0.890)	(-2.810)	(-1.097)	(-2.899)
α_1	-	-	-0.096	-0.002	-	-	-0.0098	-0.0019
			(1.830)	(-2.211)			(1.815)	(2.269)
α_2	-	-	0.021	0.026	-	-	0.0200	0.0261
			(1.467)	(2.209)			(1.505)	(2.265)
ω	0.037	0.037	0.012	0.013	0.0478	0.0449	0.0023	0.0026
	(2.164)	(3.566)	(0.777)	(0.466)	(2.894)	(3.378)	(0.399)	(-0.056)
β_1	0.391	0.376	0.236	0.269	0.6826	0.6598	0.5855	0.5903
	(1.091)	(2.607)	(2.329)	(4.221)	(3.679)	(8.688)	(3.859)	(7.639)
ϕ_1	-	-	-	-	0.2054	0.1925	0.2459	0.2216
					(2.573)	(3.948)	(2.532)	(3.836)
d	0.405	0.411	0.3121	0.318	0.5715	0.5289	0.4340	0.4350
	(1.390)	(3.180)	(5.386)	(6.682)	(3.613)	(7.061)	(4.612)	(6.839)
α_3	-	-	0.109	0.098	-	-	0.1279	0.1243
			(3.882)	(5.157)			(4.112)	(5.891)
α_4	-	-	0.004	0.001	-	-	0.0004	-0.0012
			(1.029)	(0.471)			(0.915)	(0.375)
ν	-	6.931		7.491	-	7.0341	-	7.4071
		(8.545)		(11.680)		(12.505)		(11.737)
b_3	-0.194	-0.209	-0.157	-0.182	-0.189	-0.186	-0.149	-0.146
b_4	1.829	-0.135	1.611	-0.040	1.851	-0.031	1.529	-0.168
Q(20)	48.691 ***	48.474 ***	44.554 ***	45.136 ***	46.704 ***	48.593 ***	45.938 ***	47.880 ***
$Q^2(20)$	36.617 ***	35.454 ***	33.514 ***	33.515 ***	29.237 *	32.209 ***	22.860	25.603
BDS(6)	-2.443	-1.437	-2.371	-1.214	-0.028	-0.620	-1.164	-0.060
Log Lik	-4651.6	-4561.2	-4605.1	-4527.9	-4643.4	-4555.1	-4594.7	-4521.1
Distribution		[0.000]		[0.000]		[0.000]		[0.000]
Daily effects			[0.000]	[0.000]			[0.000]	[0.000]

Note: see Table 5

Table 9 : ARFIMA(n,d,s) - FIGARCH(1,d,1) models

	DEM		FRF		GBP		YEN	
(n,d,s)	*(0,d,1)*		*(0,d,1)*		*(1,d,0)*		*(2,d,0)*	
μ	0.0009		0.0024		-0.0086		0.0010	
	(0.064)		(0.184)		(-0.813)		(0.073)	
ψ_1	-		-		-0.0495		-0.0971	
					(-2.162)		(-3.546)	
ψ_2	-		-		-		-0.0439	
							(-2.324)	
θ_1	-0.1090		-0.1035		-		-	
	(-3.777)		(-3.718)					
ζ	0.0454		0.0469		0.0202		0.0487	
	(2.042)		(2.237)		(1.134)		(2.163)	
α_1	-0.0007		-0.0070		-0.0070		0.0292	
	(-0.047)		(-0.526)		(-0.568)		(2.334)	
α_2	0.0263		0.0257		0.0105		0.0220	
	(2.360)		(2.483)		(1.075)		(2.217)	
ω	0.0044		0.0029		-0.0098		0.0001	
	(0.341)		(0.240)		(-1.064)		(0.009)	
β_1	0.5895		0.5560		0.6343		0.5320	
	(7.412)		(7.470)		(10.695)		(7.336)	
ϕ_1	0.2261		0.2369		0.3380		0.2145	
	(4.841)		(4.730)		(6.427)		(3.652)	
d	0.4341		0.4116		0.4239		0.4270	
	(4.629)		(4.951)		(7.601)		(6.914)	
α_3	0.1210		0.1142		0.0989		0.1114	
	(5.918)		(5.825)		(5.806)		(5.804)	
α_4	-0.0027		0.0001		0.0113		0.0040	
	(-0.219)		(0.011)		(1.256)		(0.406)	
ν	7.4006		6.4396		6.8740		5.5378	
	(8.864)		(9.939)		(10.512)		(11.858)	
b_3	-0.158		-0.090		0.083		-0.356	
b_4	-0.102		0.272		-0.710		1.309	
Q(20)	34.768	***	36.796	***	32.353	***	35.343	***
$Q^2(20)$	26.364	*	22.650		14.027		15.726	
BDS(6)	-0.046		0.258		0.209		1.542	
Log Lik	-4519.2		-4335.4		-4138.3		-4081.9	
LRT	[0.019]		[0.012]		[0.227]		[0.019]	

Note: asymptotic heteroskedasticity-consistent t-statistics are in parentheses and p-values of the LRT are in brakets.

Model: $\Psi(L)(1-L)^{\zeta}[r_t-(m+\alpha_1 x_{1,t}+\alpha_2 x_{2,t})]=\Theta(L)\epsilon_t$ and

$$\sigma_t^2=(\omega+\alpha_3 x_{1,t}+\alpha_4 x_{2,t})+\left\{1-[1-\beta_1 L]^{-1}(1-\phi_1 L)[1-L]^d\right\}\epsilon_t^2$$

FIGURES

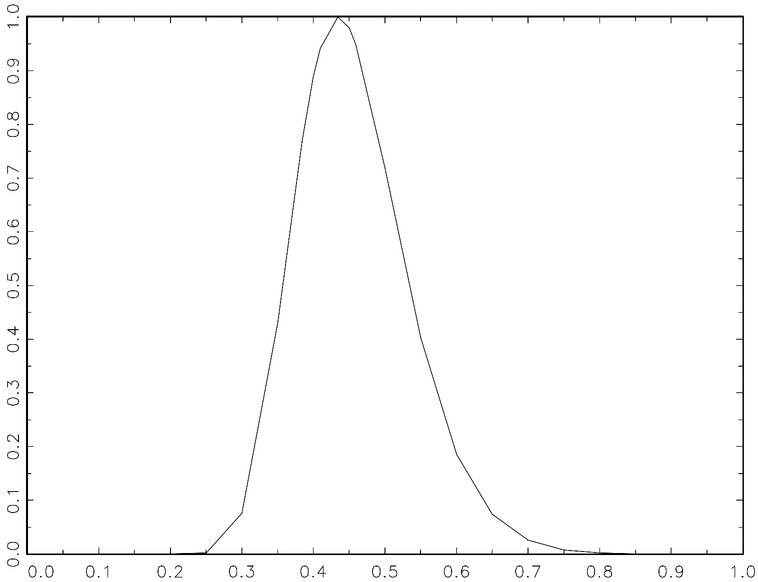

Figure 1: normed log-likelihood profile for the FIGARCH$(1, d, 1)$ model of the DEM exchange rate.

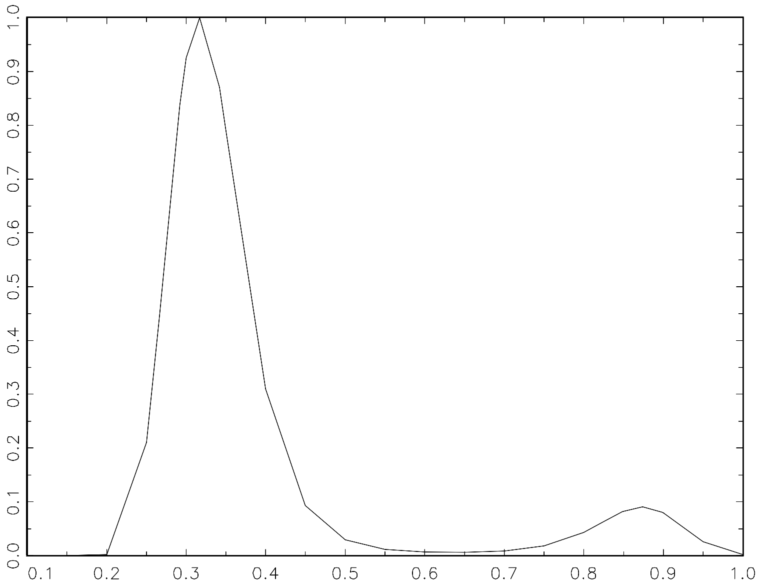

Figure 2: normed log-likelihood profile for the FIGARCH$(1, d, 0)$ model of the DEM exchange rate.

Notes

[1]See Diebold (1987).

[2]Serial correlation may be due to central bank interventions (see for example the model proposed by König and Gaab, 1982).

[3]In the case of a FIGARCH$(1, d, 0)$, $\phi_1 = 0$.

[4]Some of these sufficient conditions are nevertheless not necessary. For instance, they specify $\omega > 0$. By contrast, our estimation procedure allows ω to be negative but checks the positiveness of the conditional variance on a case-by-case basis (see Nelson and Cao, 1992).

[5]We have estimated the model with middle-week days dummies but these additional explanatory variables are in general not significant, like in Hsieh (1989), Baillie and Bollerslev (1989) .

[6]Here, in the conditional variance, it is assumed that the non-trading effect is peculiar to the day before or after the non-trading event. In other words, the initial implicit GARCH$(1, 1)$ representation is written as : $\sigma_t^2 = (\omega + \alpha_3 x_{1,t} + \alpha_4 x_{2,t})(1 - \beta_1 L)) + \mathrm{f}_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$, where f_1 is the MA_1 coefficient of the GARCH. As an alternative, it is possible to allow the non-trading day to affect through $\beta(L)$ the conditional variance for several days with a geometrically declining influence. This last specification leads however to counter-intuitive results on the one hand, and to a lower value for the log-likelihood on the other hand.

[7]The level of significance of the Box-Pierce statistics on the standardized residuals are adjusted by the number of ARMA parameters while the Box-Pierce statistics on the squared standardized residuals are adjusted by the number of GARCH parameters (see Bollerslev and Mikkelsen, 1996).

[8]For instance, the first element of this row gives the p-value of the LRT of the FIGARCH$(1, d, 0)$ model without closing days and with a Student-t distribution against the FIGARCH$(1, d, 0)$ model without closing days and with a Normal distribution.

[9]Likewise, the first p-value refers for instance to the LRT involving the FIGARCH$(1, d, 0)$ model with a Gaussian distribution and closing days effects against a FIGARCH$(1, d, 0)$ model with a Gaussian distribution and without closing days effects.

[10]As illustrated in Table 7, if one ignores the closing days effects and neglects to choose a Student-t distribution, a FIGARCH$(1, d, 0)$ model can be selected at the 1% significance level (this level is more relevant than the 5% one given the number of observations).

[11]Furthermore, if $\zeta \leq -1/2$, the process is non invertible and if $\zeta \geq 1/2$, the process is not stationary.

[12]It should be clear that the orders selection procedure has to be carried out on the complete model, i.e. to the ARFIMA-FIGARCH models in our case. Restricting the investigation to the ARFIMA part without allowing for conditional heteroscedasticity leads to different outcomes with respect to both the selected AR or MA orders and to the inference on the ζ parameter. This suggests that the allowance for a time-dependent variance is an important feature for the estimation of ARFIMA models.

[13]For computational reasons, we restrict the orders to $n \leq 2$ and $p \leq 2$. Indeed, the computation time needed to estimated ARFIMA-FIGARCH models with high MA and AR orders can be fairly important. For illustration purposes, the time needed to estimate an ARFIMA$(2, \delta, 2)$-FIGARCH$(1, d, 1)$ model for the YEN amounts to more than 12 hours (this may be more important if initial parameters values are badly chosen). Notice however that the parameters estimates are found to be quite stable across models.

[14]Unsurprizingly, the results for the GBP regarding the inference of the fractional unit root primarily depend on the selected models. Indeed, in the case of the the ARFIMA$(1,\zeta,2)$ model (selected by the Akaike and the Shibata criteria), we found ζ to be significant.

[15]Nevertheless, while intuitive, this results is worth being emphasized. Indeed, Teyssière (1997) shows that a misspecification (though a simple AR(p) model for instance) in the conditional mean can induce important biases in the conditional variance (excepted for the d parameter) even for low values of ζ.