VOLATILITY MODELS AND THEIR APPLICATIONS

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CHAPTER 1

NON-PARAMETRIC TESTS FOR INTRADAY JUMPS: IMPACT OF PERIODICITY AND MICROSTRUCTURE NOISE

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Opening, lunch and closing of financial markets induce a periodic component in the volatility of high-frequency returns. Accounting for this periodicity in the implementation of non-parametric intraday jump tests is important to avoid a size distortion. We review this topic and extend it by explicitly taking microstructure noise into account in the analysis of the size and power of intraday jump test statistics.

1.1 INTRODUCTION

The occurrence of jumps is of particular interest in many practical financial applications. For example, jumps have distinctly different implications for the valuation of derivatives ([31];[32]), risk measurement and management ([21]), as well as asset allocation ([23]).

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To learn about the features of jump arrivals and their associated market information, a variety of formal tests have been developed to identify the presence of jumps, most of them being typically designed for the analysis of low frequency data (see e.g. [1], [17], [24] and [39], among others). However, the most natural and direct way to learn about jumps is by studying high frequency or intraday data.

The earliest contributions in the identification of jumps using intraday data include [8] and [9]) who developed a jump robust measure of the daily integrated variance called realized bipower variation. They provide a convenient non-parametric framework that uses the difference between the realized variance and the bipower variation to measure the contribution of jumps to total return variation and for classifying days on which jumps have or have not occurred.

However, for many financial applications, the presence of jumps needs to be detected over very short time intervals, such as 5 minutes. Over these intervals, there are often not enough observations to test for jumps using the difference between an estimator of the total return variance and a jump-robust variance estimate. Recently, [4] and [28] independently developed an alternative non-parametric test which tests directly whether a given high-frequency return has a jump component. The proposed jump test statistic in [4] and [28] is the same and corresponds to the absolute high-frequency return, standardized by a robust estimate of its volatility. As such, this test makes it possible to identify the exact timing of the jump and to characterize the jump size distribution and stochastic jump intensity.

The non-parametric jump test of [4] and [28] (or extensions of it such as in [27]) has enabled researchers to study the relation between jumps and various financial and macroeconomic variables (see e.g. [10] and [38] among others). For instance, [36] investigate the market variance risk-premium and [26] look at the relation between macroeconomic announcements and jumps.

It is common to implement the jump test of [4] and [28] using a rescaled version of a jump robust estimate of the daily integrated variance as an approximation for the variance of the high-frequency return. The approximation error is small when the volatility is approximately constant over the day. As noted by [4] and [11], among others, this assumption is at odds with the overwhelming evidence of a strong periodic pattern in intraday volatility. [13] show that accounting for periodicity greatly improves the accuracy of intraday jump detection methods. It increases the power to detect the relatively small jumps occurring at times for which volatility is periodically low and reduces the number of spurious jump detections at times of periodically high volatility.

[4], [13] and [28] assume that the log-prices are observed without measurement error. It is however more realistic to consider that the logarithm of the recorded asset price is actually the sum of the logarithm of the so-called “efficient” price and a noise component that is induced by microstructure frictions. [2] divide the sources of microstructure noise in three groups: (i) frictions inherent in the trading process such as bid-ask bounces, discreteness of price changes and rounding; (ii) informational effects such as the gradual response of prices to a block trade or inventory control effects and (iii) measurement or data recording errors such as prices entered as zero or misplaced decimal points. In this chapter we analyze the sensitivity of intraday

jump tests to the presence of both intraday periodicity and microstructure noise in the data.

The effect of microstructure noise on estimators of the integrated variance and covariance has been widely studied, see [5], [6], [29], [34] and [40], among others. Accounting for microstructure noise in non-parametric jump detection is recently studied by [12] and[15].

The goal of this chapter is to illustrate the effect of periodicity and microstructure noise on the non-parametric detection of intraday jumps, emphasizing the interaction between both effects. We proceed as follows. Sections 1.2 and 1.3 state the model settings and describe the jump statistic. We then compare the different implementations of the test on simulated data and stock price data in Sections 1.4 and 1.5. Section 1.6 concludes the chapter.

1.2 MODEL

In essence, a price jump is a significant discontinuity in the price process. To define what is meant by "significant discontinuity", we need an underlying price model. We suppose that the log-price process $\{X_t\}$ is a combination of a latent log-price process $\{\tilde{X}_t\}$ and a noise process $\{\varepsilon_t^{\mathrm{x}}\}$:

$$X_t = \tilde{X}_t + \varepsilon_t^{\mathrm{x}}. \tag{1.1}$$

The latent process $\tilde{X}_t$ is supposed to be a Brownian semimartingale process with finite activity jumps. This means that it can be decomposed into a drift, a stochastic volatility diffusion and a jump component:

$$d\tilde{X}_t = \mu_t dt + \sigma_t dW_t + K_t dq_t, \tag{1.2}$$

where μ_t is the drift term with a continuous and locally finite variation sample path, σ_t is a strictly positive spot volatility process and W_t is a standard Brownian motion. The component $K_t dq_t$ refers to the pure jump component, where $dq_t = 1$ if there is a jump at time t and 0 otherwise, and K_t represents the jump size. The jump process is supposed to be independent of the diffusion process and to have finite activity, which means that the number of jumps over any interval of time is finite with probability 1.

For applications, one is often interested in price jump detection at a given frequency Δ, such as 2 minutes [14] or 5 minutes [26, 28]. This requires to compute returns on an equispaced calendar time grid, with last price interpolation. We normalize the length of one day to unity and focus on the day $[0, 1]$. Denote $r_{i,\Delta} = X_{i\Delta} - X_{(i-1)\Delta}$ the corresponding returns sampled at the frequency Δ, for $i = 1, \ldots, \lfloor 1/\Delta \rfloor$.[1] If there is no jump in the interval $[(i-1)\Delta, i\Delta]$, we have that:

$$r_{i,\Delta} = \tilde{X}_{i\Delta} - \tilde{X}_{(i-1)\Delta} + \varepsilon_{i\Delta}^{\mathrm{x}} - \varepsilon_{(i-1)\Delta}^{\mathrm{x}} = \int_{(i-1)\Delta}^{i\Delta} \mu_s ds + \int_{(i-1)\Delta}^{i\Delta} \sigma_s dW_s + \varepsilon_{i\Delta}^{\mathrm{x}} - \varepsilon_{(i-1)\Delta}^{\mathrm{x}}.$$

[1] The function $\lfloor \cdot \rfloor$ returns the largest integer less than or equal to its argument.

Throughout, we will be operating with sufficiently high-frequency return series such that the mean process can be safely ignored.

We impose two high-level assumptions on the microstructure noise: (i) $\varepsilon_t^{\mathrm{x}}$ is independent of the X process and (ii) $\varepsilon_{\mathrm{x}} \overset{i.i.d.}{\sim} N(0, \sigma^2_{\varepsilon_{\mathrm{x}}})$. Under these assumptions, we have that for Δ sufficiently small, the high-frequency return $r_{i,\Delta}$ is still normally distributed in the presence of noise (but absence of jumps), with variance equal to the sum of the integrated variance over $[(i-1)\Delta, i\Delta]$ and two times the noise variance:

$$\frac{r_{i,\Delta}}{\left[\int_{(i-1)\Delta}^{i\Delta} \sigma_s^2 ds + 2\sigma^2_{\varepsilon_{\mathrm{x}}}\right]^{\frac{1}{2}}} \sim N(0,1). \tag{1.3}$$

The result in (1.3) suggests to detect the presence of a jump in $r_{i,\Delta}$ whenever the standardized high-frequency return exceeds an extreme quantile of the normal distribution. This requires to estimate $\int_{(i-1)\Delta}^{i\Delta} \sigma_s^2 ds$. One possible avenue is to use local volatility estimates based on moving windows and kernel weights, as in [25], [30], [35] and [28].

An alternative is to impose more structure on the spot volatility process. [3] and [37] assume that the local volatility equals the daily volatility corrected for the intraday periodic pattern:

$$\int_{(i-1)\Delta}^{i\Delta} \sigma_s^2 ds = f^2_{i,\Delta} \Delta \int_0^1 \sigma_s^2 ds. \tag{1.4}$$

The periodicity factor $f_{i,\Delta}$ is supposed to be a deterministic function of periodic variables such as the time of the day, the day of the week and macroeconomic news announcements. The decomposition (1.4) is unique under the standardization condition that the squared periodicity factor has mean one over the day.

1.3 PRICE JUMP DETECTION METHOD

In this section we first discuss the different building blocks for the construction of a jump detection method under the assumption that the high-frequency returns $r_{i,\Delta}$ follow the model given by (1.3) and (1.4). We need an estimator of the noise variance, the integrated variance and the periodicity factor. Next we present several jump test statistics based on these estimates. Finally, we discuss the choice of the critical value.

Estimation of the noise variance. Estimators of the noise variance $\sigma^2_{\varepsilon_{\mathrm{x}}}$ are typically based on prices sampled in transaction time. Suppose the log-price process is observed at time points $0 \leq t_1 < t_2 < ... < t_{n+1} \leq 1$, yielding the observations $X_{t_1}, X_{t_2}, \ldots, X_{t_{n+1}}$. [41] show that the noise variance is consistently estimated by $1/(2n)$ times the difference between the realized variance computed on the tick-by-tick log-price changes and the two time scale realized volatility (TSRV):

$$\hat{\sigma}^2_{\varepsilon_{\mathrm{x}}} = \frac{1}{2n}([X,X]^{(all)} - \mathrm{TSRV}) \xrightarrow{p} \sigma^2_{\varepsilon_{\mathrm{x}}}. \tag{1.5}$$

Let us briefly discuss the TSRV. It has two components, namely the realized variance computed on the tick-by-tick returns and averaged realized variance computed on returns sampled at a lower time scale. The realized variance calculated on the tick-by-tick returns is

$$[X,X]^{(all)} = \sum_{i=1}^{n} (X_{t_{i+1}} - X_{t_i})^2.$$

The TSRV of [41] is based on partitioning the whole sample into K subsamples, with K an integer. Denote the average K subsampled realized variance as:

$$[X,X]^{(avg)} = \frac{1}{K} \sum_{i=1}^{n-K+1} \left(X_{t_{i+K}} - X_{t_i}\right)^2.$$

The TSRV is defined as the difference between the averaged realized variance computed over K step apart subsampled observations and the adjusted realized variance computed using all observations:

$$\text{TSRV} = (1 - \frac{\overline{n}}{n})^{-1}([X,X]^{(avg)} - \frac{\overline{n}}{n}[X,X]^{(all)}), \tag{1.6}$$

where $\overline{n} = (n - K + 1)/K$. Taking the difference between $[X,X]^{(avg)}$ and $\frac{\overline{n}}{n}[X,X]^{(all)}$ cancels the effect of the microstructure noise. The factor $(1 - \overline{n}/n)^{-1}$ is a coefficient to adjust for finite sample bias.

Robust estimators of the integrated variance. Several propositions have been made over the years to estimate the integrated variance (see [33] for a general review). To guarantee a high power of the jump tests, we need to restrict ourselves to jump robust estimators, otherwise jumps will inflate the variance estimate and deflate the jump statistic based on (1.3). If there are multiple jumps within a day, this might lead to a failure to detect the smaller jumps, a phenomenon which is called “outlier masking” in the robustness literature (see e.g. [20]). For simplicity, and like in [4], [28] and [13], we estimate the integrated variance using returns sampled at the same frequency as the one at which jumps need to be detected, namely Δ. In empirical work, the realized bipower variation of [8] is especially popular:

$$\text{BPV}_\Delta = \frac{\pi}{2}\frac{M_\Delta}{M_\Delta - 1}\sum_{i=2}^{M_\Delta} |r_{i,\Delta}||r_{i-1,\Delta}|, \tag{1.7}$$

with $M_\Delta = \lfloor 1/\Delta \rfloor$.

In the presence of microstructure noise, the BPV estimates the integrated variance plus the noise variation in the returns used to compute the integrated variance (i.e. $\int_0^1 \sigma_s^2 ds + 2M_\Delta \sigma_{\varepsilon_X}^2$). This bias can be easily corrected for by using the following bias adjusted version of the BPV:

$$\text{BPV}^*_\Delta = \text{BPV}_\Delta - 2M_\Delta \hat{\sigma}^2_{\varepsilon_X}. \tag{1.8}$$

This estimator is directly related to the class of two time scale realized variance estimators proposed by [41]. Note that the lower the sampling frequency is, the smaller is M_Δ and thus also the bias correction in (1.8). Alternatively, one could use the modulated bipower variation described in [34], or the robust TSRV of [15].

Periodicity estimation. Under the model (1.3) and (1.4), but without microstructure noise, the periodicity factor corresponds to the standard deviation of the standardized returns $\overline{r}_{i,\Delta} = r_{i,\Delta}/(\widehat{IV}/M_\Delta)^{1/2}$, with $\widehat{IV}$ an estimator of the integrated variance such as the bipower variation in (1.7). [37] propose the following estimation procedure for f_i. Collect first all the standardized returns having standard deviation f_i. Denote these: $\overline{r}_{1,i}, \ldots, \overline{r}_{n_i,i}$, with n_i the number of standardized returns having standard deviation f_i. Compute then their standard deviation, i.e. $\mathrm{SD}_{i,\Delta} = \sqrt{\frac{1}{n_i}\sum_{j=1}^{n_i} \overline{r}_{j,i}^2}$. Finally, their estimator is a standardized version of these periodicity estimates:

$$\hat{f}_{i,\Delta}^{\mathrm{SD}} = \frac{\mathrm{SD}_{i,\Delta}}{\sqrt{\frac{1}{M_\Delta}\sum_{j=1}^{M_\Delta} \mathrm{SD}_{j,\Delta}^2}}. \tag{1.9}$$

[3] show that more efficient estimates can be obtained if the whole time series dimension of the data is used for the estimation of the periodicity process. They consider the regression equation

$$\log|\overline{r}_{i,\Delta}| - c = \log f_{i,\Delta} + \varepsilon_{i,\Delta}, \tag{1.10}$$

where the error term $\varepsilon_{i,\Delta}$ is i.i.d. distributed with mean zero and having the density function of the centered absolute value of the log of a standard normal random variable, and $c = -0.63518$. [3] then propose to model $\log f_{i,\Delta}$ as a linear function of a vector of variables x_i (such as sinusoid and polynomial transformations of the time of the day)

$$\log f_{i,\Delta} = x_{i,\Delta}'\theta_*, \tag{1.11}$$

with θ_* the true parameter value. [13] propose a jump robust regression estimator for this model called the "truncated maximum likelihood" periodicity estimate. Call the resulting periodicity estimates $\hat{f}_{i,\Delta}$. Figure 1.1 plots this estimate for the GE stock based on the 2-minute return data in 2008, extracted from the NYSE trades and quotes database.[2] Note the clear U-shape induced by the opening, lunch and closing of the stock market. This periodicity estimate is close to the pattern one would obtain using 5-minute returns and therefore seems to be robust to microstructure noise.[3]

[2]The periodicity factor is estimated using the default implementation in the R package RTAQ [19].

[3]The periodicity estimates mentioned above assume that, in the absence of jumps, the standardized return $r_{i,\Delta}(IV/M_\Delta)^{-1/2}$ is normally distributed with variance f_i^2. In presence of microstructure noise, its variance is however $f_i^2 + 2M_\Delta\sigma_\varepsilon^2/IV$. It seems thus that the smaller the sampling frequency, the larger may be the bias introduced by the noise in the periodicity estimation. An analysis of this bias and the development of a periodicity estimate that is robust to microstructure noise and jumps is left for further research. Of course, given the smoothness of the periodicity pattern for equity and foreign exchange data, the parameter θ_* can always be estimated at a lower frequency than the one at which jumps need to be detected.

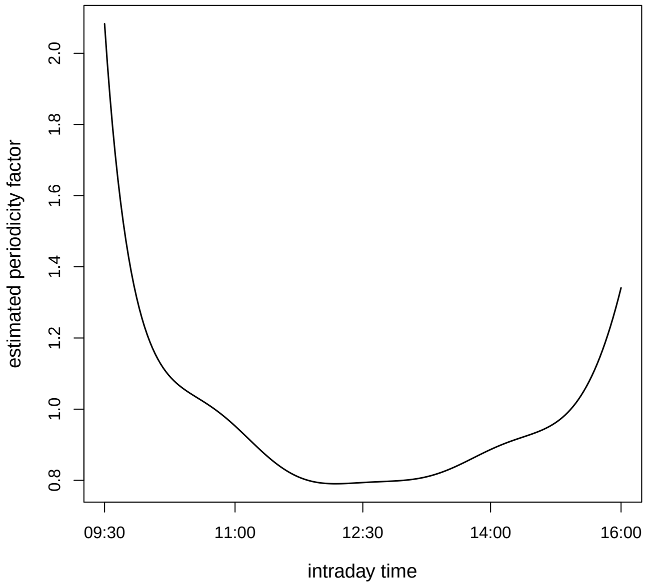

Figure 1.1 Intraday plot of estimated 2-minute intraday periodicity factors for General Electric.

Jump test statistics. The model in (1.3) and (1.4) suggests three statistics. The first one implicitly assumes the absence of an intraday volatility pattern and microstructure noise, namely:

$$I_{i,\Delta} = \frac{|r_{i,\Delta}|}{\sqrt{M_\Delta^{-1}\mathrm{BPV}_\Delta}}. \tag{1.12}$$

This test statistic is closely related to the original test proposed by [4] and [28].[4] Note that the denominator in $I_{i,\Delta}$ estimates $\int_{(i-1)\Delta}^{i\Delta} \sigma_s^2 ds + 2\sigma_{\varepsilon_X}^2$, while the variance of $r_{i,\Delta}$ is $f_{i,\Delta}^2 \int_{(i-1)\Delta}^{i\Delta} \sigma_s^2 ds + 2\sigma_{\varepsilon_X}^2$ in the absence of jumps. It follows that jump detection based on the I-test in (1.12) overdetects (resp. underdetects) jumps when volatility is periodically high (low).

[4]Alternatively, sliding local windows centered around $r_{i,\Delta}$ could be used for the estimation of $\int_{(i-1)\Delta}^{i\Delta} \sigma_s^2 ds$. If returns are sampled at frequencies of one hour, 30 minutes, 15 minutes or 5 minutes, [28] recommend to use local windows containing 78, 110, 156 or 270 observations, respectively. These numbers correspond to the smallest number of observations for which jumps will have a negligible effect on the realized bipower variation. For simplicity, we always take local windows of one day.

[13] propose the following adjustment:

$$J_{i,\Delta} = \frac{|r_{i,\Delta}|}{\hat{f}_{i,\Delta}\sqrt{M_{\Delta}^{-1}\mathrm{BPV}_{\Delta}}}. \tag{1.13}$$

This version of the test implicitly assumes that the microstructure noise follows the same periodic pattern as the spot volatility of the underlying efficient price process. This is inconsistent with the assumption that the variance of the microstructure noise is constant over the day. For this reason, we also consider a third statistic, which is consistent with the model in (1.3) and (1.4)

$$Z_{i,\Delta} = \frac{|r_{i,\Delta}|}{\sqrt{\hat{f}_{i,\Delta}^{2} M_{\Delta}^{-1}\mathrm{BPV}_{\Delta}^{*} + 2\hat{\sigma}_{\varepsilon_{\mathrm{X}}}^{2}}}. \tag{1.14}$$

Note the use of $\mathrm{BPV}_{\Delta}^{*}$ instead of BPV_{Δ}.

Critical value. The presence of a jump in $r_{i,\Delta}$ is detected when the jump test statistic exceeds a critical value. There are several possibilities to choose this threshold. A straightforward jump detection rule is that return $r_{i,\Delta}$ is affected by a jump if the test statistic exceeds the $1-\alpha/2$ quantile of the standard Gaussian distribution. This rule has a probability of type I error (detect that $r_{i,\Delta}$ is affected by jumps, if in reality $r_{i,\Delta}$ is not affected by jumps) equal to α. Its disadvantage is that the expected number of false positives becomes large. For example, with $M_{\Delta} = 195$ intraday returns per day and $\alpha = 0.01$, one expects to detect about $0.01 \cdot 195 \approx 2$ jumps per day, even if no single jump has occurred. [28] call these false positives “spurious jump detections”.

For this reason, [4] and [28] propose to control the size of multiple jump detection tests. [4] use a Šidák correction to control for the number of spurious jumps detected per day. Under this approach, the rejection threshold is given by the $[1-(1-\alpha)^{\Delta}]/2$ quantile of the Gaussian distribution. [28] use a different way to control for the size of multiple tests. They advocate to use the extreme value theory result that the maximum of L i.i.d. realizations of the absolute value of a standard normal random variable is asymptotically (for $L \to \infty$) Gumbel distributed. More specifically, in the absence of jumps, the probability that the maximum of any set of L test statistics exceeds

$$g_{L,\alpha} = -\log(-\log(1-\alpha))b_L + c_L, \tag{1.15}$$

with $b_L = 1/\sqrt{2\log L}$ and $c_L = (2\log L)^{1/2} - [\log \pi + \log(\log L)]/[2(2\log L)^{1/2}]$, is about α. All returns for which the test statistic exceeds this critical value $g_{L,\alpha}$ are then declared as being affected by jumps. In the sequel of the paper, we set $L = M_{\Delta} = 195$. This corresponds to testing for the joint null hypothesis of no jumps over one day, for a market that is open 6.5 hours a day with returns sampled every two minutes. We set $\alpha = 1\%$. For these values of L, Δ and α, the rejection threshold used in [4] and [28] equals approximately 4.049 and 4.232, respectively.

1.4 SIMULATION STUDY

This section consists of two parts. First, we illustrate the intraday differences in the values of the I, J and Z statistics defined in Section 1.3 and discuss how these differences affect the size and power of the test. Second, we apply the three test statistics to a simulated price process and investigate their properties on a daily and intraday level.

1.4.1 Intraday differences in the value of the test statistics

Through stylized examples, we illustrate here the intraday differences between I, J and Z. This graphical analysis will shed light on the differences in size and power of testing for jumps using these three statistics. We make abstraction of estimation error by assuming that $\int_0^1 \sigma^2(s)ds = \text{BPV}^*_\Delta = 1$, $\sigma^2_{\varepsilon_\text{X}} = \hat{\sigma}^2_{\varepsilon_\text{X}} = 0.001$ and the periodicity factor $f_{i,\Delta} = \hat{f}_{i,\Delta}$ is as shown in Figure 1.1. Returns are sampled every two minutes.[5]

We first consider the case where the high-frequency return $r_{i,\Delta}$ is generated by the model (1.3) and (1.4), but is relatively large, i.e. we set $|r_{i,\Delta}|$ to 3.719 (the 99.99% quantile of the Gaussian distribution) times the volatility $\sqrt{f^2_{i,\Delta}\Delta \int_0^1 \sigma^2(s)ds + 2\sigma^2_{\varepsilon_\text{X}}}$. Because of the periodicity component, $|r_{i,\Delta}|$ fluctuates throughout the day, while the threshold remains constant. Figure 1.2 reports the values of $I_{i,\Delta}$, $J_{i,\Delta}$ and $Z_{i,\Delta}$ during one day. The value of $Z_{i,\Delta}$ obviously remains constant at 3.719 and serves as the benchmark. The value of $I_{i,\Delta}$ fluctuates substantially throughout the day, because the periodicity pattern is ignored. Note that if we use the threshold of 4.232, we detect all returns between 09:30 EST and 10:06 EST as spurious jumps based on $I_{i,\Delta}$. The value of $J_{i,\Delta}$ also fluctuates throughout the day. Instead of keeping the noise variance component constant at $2\sigma^2_{\varepsilon_\text{X}}$, it implicitly sets it to $2f^2_{i,\Delta}\sigma^2_{\varepsilon_\text{X}}$. It follows, as can be seen in Figure 1.2, that spurious jump detection will increase around noon.

A return takes extreme values when it is affected by a jump. These are the returns we would like to detect. We consider a large return whose diffusion component is zero: $|r_{i,\Delta}| = 0.35$, for $i = 1, \ldots, M_\Delta$. Figure 1.3 plots the intraday evolution of the corresponding $I_{i,\Delta}$, $J_{i,\Delta}$ and $Z_{i,\Delta}$ test statistics. While $I_{i,\Delta} = 4.15$, for all $i = 1, \ldots, 195$, the value of $J_{i,\Delta}$ and $Z_{i,\Delta}$ fluctuates throughout the day because these test statistics take into account the periodicity pattern. Note that $J_{i,\Delta}$ detects more jumps than $Z_{i,\Delta}$ during the middle of a trading day and less during the opening and closing period. In contrast to $Z_{i,\Delta}$, $J_{i,\Delta}$ does not take into account the noise variance separately, and hence overestimates (underestimates) the return volatility when $f_{i,\Delta} > 1$ ($f_{i,\Delta} < 1$), which explains this observation.

These stylized examples showed that the differences in the value of the I, J and Z test statistics can thus be substantial at the intraday level. Under the model in (1.3)

[5] We assume that the market is open 6.5 hours a day, resulting in 195 returns a day.

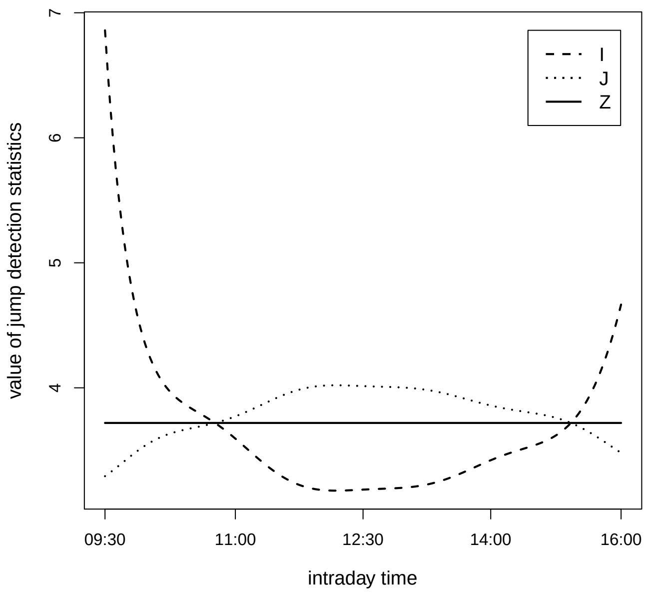

Figure 1.2 Effect of periodicity on the value of the I, J and Z jump test statistics when $r_{i,\Delta} = 3.719\sqrt{f_{i,\Delta}^2 \Delta \int_0^1 \sigma^2(s)ds + 2\sigma_{\varepsilon_X}^2}$. The critical value for jump detection is 4.232.

and (1.4), we advise to use $Z_{i,\Delta}$ to detect jumps, as it correctly takes into account the noise variance and periodicity pattern.

1.4.2 Comparison of size and power

The previous subsection indicated that the differences between the size and power of the I, J and Z test statistics in (1.12)-(1.14) can be substantial on an intraday scale. This does not necessarily imply big differences on an aggregated basis, since the overdetection of jumps during a certain part of the day can be offset by the underdetection during another part of the day. In the next part, we evaluate the effective size and power of the three test statistics for a general log-price process with stochastic volatility, intraday periodicity and finite activity jumps. We first describe the simulation setup and then discuss the results.

Simulation setup. We simulate a log-price process for 1000 days, with the time interval of one day standardized to $t \in [0, 1]$. The log-price series are simulated from

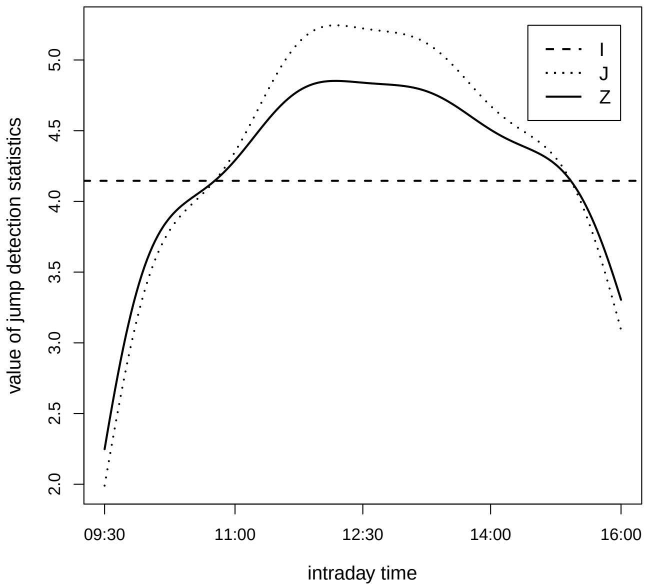

Figure 1.3 Effect of periodicity on the value of the I, J and Z jump test statistics when $|r_{i,\Delta}| = 0.35$. The critical value for jump detection is 4.232.

the jump diffusion model in [22]:

$$\begin{cases} X_t = \tilde{X}_t + \varepsilon_t^{\mathrm{x}} \\ d\tilde{X}_t = \mu_{\mathrm{x}} dt + f_t \exp(\beta_0 + \beta_1 v_t^{\mathrm{x}}) dW_t^{\mathrm{x}} + dJ_t^{\mathrm{x}} \\ dv_t^{\mathrm{x}} = \alpha_v v_t^{\mathrm{x}} dt + dB_t^{\mathrm{x}} \\ Corr(dW_{\mathrm{x}}, dB_{\mathrm{x}}) = \gamma, \end{cases} \tag{1.16}$$

where γ is the leverage correlation, v_t^{x} is a stochastic volatility process, f_t accounts for the intraday periodicity, and J_t^{x} is a compound Poisson process with jump intensity λ. For the diffusion, the parameters $(\mu_{\mathrm{x}}, \beta_0, \beta_1, \gamma, \lambda)$ are set to $(0.03, -5/16, 1/8, -0.3, 2)$. The jump tests in (1.12)-(1.14) implicitly proxy the stochastic volatility component by a constant (the daily integrated variance). To study the effect of this approximation on the size and power of the tests, we consider three types of spot volatility dynamics: a very persistent stochastic volatility ($\alpha_{v_1} = -0.00137$) with and without a periodic pattern and a less persistent stochastic volatility process ($\alpha_{v_2} = -1.386$) without periodicity. The absence of a periodic pattern means that $f_t = 1$. In the presence of periodicity, we implement the periodicity factor estimated for the GE stock during 2008, which is shown in Figure 1.1. The initial value of v_t^{x} for each day is drawn from a normal distribution

$N(0, (-\alpha_{v_1} - \alpha_{v_2})^{-1})$. The size of the jumps is modeled as the product between the realization of a uniformly distributed random variable on $([-2, -1] \cup [1, 2])/\sqrt{2\lambda}$ and the mean value of the spot volatility process of that day. We assume the noise $\varepsilon_t^{\mathrm{x}} \overset{i.i.d}{\sim} N(0, \sigma^2_{\varepsilon_{\mathrm{x}}})$. We consider a low noise variance of $\sigma^2_{\varepsilon_{\mathrm{x}}} = 0.001$ (corresponding to a noise-to-signal ratio of approximately 0.0014, which is of the same order as the values reported in Table 1 of [18] for S&P 500 stocks in 2006) and a high noise variance of $\sigma^2_{\varepsilon_{\mathrm{x}}} = 0.005$.

The diffusion part of the model is simulated with an increment of 1 second per tick using the Euler Scheme. We use independent Poisson sampling schemes such that the inter transaction times are exponentially distributed with on average one transaction every 5 seconds. We align the price series to a regular 2-minute grid, using the previous tick approach.

Results. Table 1.1 reports the "effective size" and "effective power" of the I, J and Z statistics. These effective size and power statistics correspond to the proportion of spuriously detected jumps and the proportion of actual jumps that have been detected with success.

A jump component is detected in a return when the test statistic exceeds 4.2322. Under the model (1.3) and (1.4), we expect therefore a size of $2(1 - \Phi(4.232)) \approx 2.32e - 5$. Because we estimate the volatility of the deseasonalized returns with the daily integrated variance, the actual effective size is slightly higher in presence of time-varying stochastic volatility. To see the effect of periodicity on the size and power of the jump test statistics, we compute the size and power not only as an aggregate over the entire day, but also for the intraday times where the periodicity factor is the highest (market opening between 9:30 and 10:30 EST and market closing between 15:00 and 16:00 EST) and the lowest (lunch time between 12:00 and 13:00 EST).

Consider first the case of no microstructure noise in columns 1-3 of Table 1.1. Note first that when the spot volatility is highly persistent and shows no periodicity, the size of the test ($2.8e - 5$) is the closest to the one expected under the model (1.3) and (1.4). In absence of periodicity in volatility, the size and the power of the I, J and Z test statistics are very similar. In the third column, we allow for periodicity and see that the size of the I-statistic ($5.70e - 5$) is too high. Moreover, it detects many more spurious jumps at times of high periodicity (size of $9.93e - 5$) than at times of low periodicity (size of $4.04e - 5$). This is not the case for the J and Z statistics where the size is similar for all intraday periods. Regarding the power of the tests, we see in columns 1-3 that the choice of test statistics has little influence. The power is the highest when the volatility process is persistent.

Consider now the case with microstructure noise. The numerator of the I, J and Z statistic is then the square of the log-difference in the efficient price plus the noise terms. For the contamination with the small microstructure noise, there is little significant effect on the size and power of the jump test statistics. The presence of large noise, however, reduces significantly the power of the jump test statistics. From the last column, it seems that in the joint presence of a large noise variance and a

Table 1.1 Effective size and power of the I, J and Z jump tests with rejection threshold equal to 4.232.

Noise variance $\sigma^2_{\varepsilon_X}$ Periodicity f_t Persistence of ν_t^X		zero no high	zero no low	zero yes high	small no high	small no low	small yes high	large no high	large no low	large yes high
						Size × 10000				
I	9:30-16:00	2.80	3.52	5.70	2.80	3.32	3.47	3.16	3.68	2.28
	9:30-10:30, 15:00-16:00	2.69	3.20	9.93	2.86	2.19	4.71	3.37	3.53	3.03
	12:00-13:30	2.47	2.47	4.04	2.25	3.14	3.37	2.25	2.92	2.47
J	9:30-16:00	2.80	3.52	3.68	2.80	3.32	2.95	3.16	3.68	2.33
	9:30-10:30, 15:00-16:00	2.69	3.20	3.20	2.86	2.19	3.03	3.37	3.53	2.86
	12:00-13:30	2.47	2.47	4.27	2.25	3.14	3.37	2.25	2.92	2.70
Z	9:30-16:00	2.80	3.52	3.63	2.80	3.32	3.00	3.16	3.68	2.28
	9:30-10:30, 15:00-16:00	2.69	3.20	3.20	2.86	2.19	3.20	3.37	3.53	3.03
	12:00-13:30	2.47	2.47	4.04	2.25	3.14	3.37	2.25	2.92	2.47
						Power × 100				
I	9:30-16:00	97.11	96.28	96.15	97.11	96.28	96.20	92.02	91.25	77.73
	9:30-10:30, 15:00-16:00	97.08	96.50	96.22	95.98	97.41	95.85	90.13	91.65	87.33
	12:00-13:30	97.07	97.30	95.90	97.12	96.53	95.19	91.90	90.87	64.47
J	9:30-16:00	97.11	96.28	96.15	97.11	96.28	96.67	91.97	91.15	79.24
	9:30-10:30, 15:00-16:00	97.08	96.50	96.22	95.98	97.41	95.66	90.03	91.32	86.45
	12:00-13:30	97.07	97.30	95.90	97.12	96.53	96.78	91.90	90.87	68.87
Z	9:30-16:00	97.11	96.28	96.15	97.11	96.28	96.62	92.02	91.15	78.10
	9:30-10:30, 15:00-16:00	97.08	96.50	96.22	95.98	97.41	95.66	90.13	91.32	86.77
	12:00-13:30	97.07	97.30	95.90	97.12	96.53	96.57	91.90	90.87	65.61

strong periodic pattern in volatility, the I, J and Z statistics all have a substantial higher power when periodicity is high than when periodicity is low.

1.5 COMPARISON ON NYSE STOCK PRICES

The effect of accounting for periodicity in intraday jump detection of foreign exchange data was studied by [13]. They find that it reduces the number of jumps detected at times of periodically high volatility, on the one hand, and leads to a significant increase in the number of jumps detected at times of periodically low volatility, on the other hand. Here we extend this finding to jumps detected in stock prices.

Our data is from the New York Stock Exchange Trades and Quotes database. The sample contains 624 trading days ranging from July 2, 2007 to December 31, 2009.[6] We consider 27 Dow Jones Industrial Average constituents at the beginning

[6]We removed half trading days from our sample, namely 2007-07-03, 2007-11-23, 2007-12-24, 2008-07-03, 2008-11-28, 2008-12-24, 2009-11-27, 2009-12-24. Prior to the analysis, the data is cleaned using the step-by-step procedure proposed in [7] and [16], and implemented in the R package RTAQ [19].

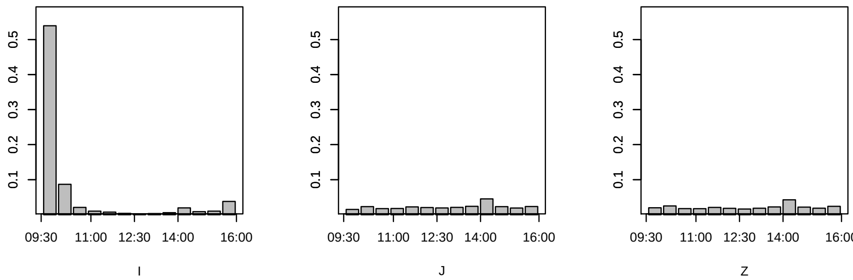

Figure 1.4 Intraday plot of the number of 2-minute intervals with jumps per 30 minutes, averaged over 27 Dow Jones Industrial Average stocks and the 624 days in the sample.

of 2008.[7] The trading session runs from 9:30 EST until 16:00 EST (390 minutes). We force these price series to a regular 2-minute grid by previous tick interpolation. We compare the average total number of 2-minute jumps detected using the I, J and Z statistics in (1.12)-(1.14) for each 30-minute interval of a trading day.

Figure 1.4 plots the average (over the 624 days and 27 stocks in the sample) number of detected jumps (in 2-minute returns) per 30-minute interval. The number of detected jumps can take values between 0 (no jumps detected in the 30-minute interval) and 15 (every 2-minute return in the 30-minute interval has a jump). Note first the U-shape in the intraday plot of number of jump detections using the I-statistic. This is in contrast with the rather uniform distribution of jump detections based on the J and Z statistics which correct for the intraday periodicity. We clearly see that during opening hours the I statistic detects more jumps than the J and Z statistics. The biggest difference is observed for the first 30-minute interval, where the I statistic detects about 1 jump per stock every 2 days on average, while the J and Z statistics only detect on average about 1 jump per stock every 66 and 50 days, respectively. When averaged over the entire day, we find that the I test statistic detects more than twice as many jumps than the J and Z statistics.

The pairwise percentage differences between the three tests are plotted in Figure 1.5. The left, central and right plot compare the percentage difference of using I vs J, I vs Z and Z vs J. In all cases, we observe a U-shaped pattern, similar to the periodicity pattern shown in Figure 1.1. Because of ignoring the intraday periodicity pattern, I detects up to 35 times more jumps during the opening period and up to 80% less jumps during noon, than the two other test statistics. Let us now focus on the

[7]Tickers of the stocks in the sample are: AA, AXP, BA, C, CAT, DD, DIS, GE, HD, HON, HPQ, IBM, INTC, JNJ, JPM, KO, MCD, MMM, MO, MRK, MSFT, PFE, PG, UTX, VZ, WMT, XOM. Because of too many missing observations, AIG, GM and T were removed from our sample.

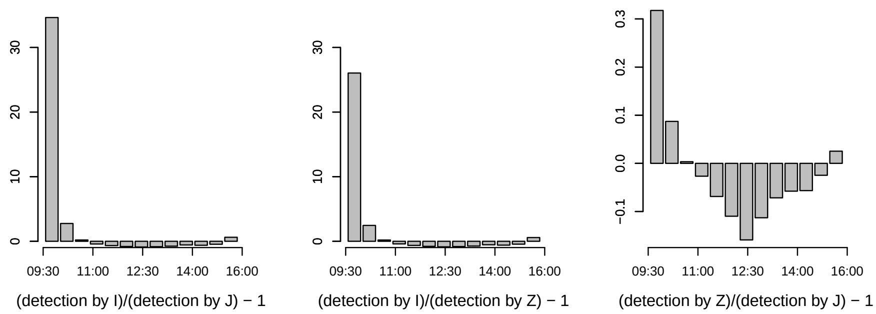

Figure 1.5 Pairwise differences of number of jumps detected by two test statistics.

comparison between Z and J. Since we find smaller differences here, it is reasonable to argue that the empirical impact of taking periodicity into account is larger than the impact of separating the continuous and noise variance in the denominator of the test statistic. The J statistic falsely imposes a periodic pattern on the noise variance, while the Z statistic keeps it constant over the day. As one can expect, shifting from J to Z, we detect more jumps during the opening period and less jumps during noon.

1.6 CONCLUSION

In this chapter we review the literature on non-parametric tests for jumps in high-frequency price series. High-frequency financial returns are typically affected by (i) microstructure noise and (ii) a U-shaped periodic variation of intraday volatility. We discuss and illustrate the impact these two features have on the jump detection statistic of [4] and [28]. Ignoring periodicity is shown to induce large size and power distortions of the jump detection statistic at the intraday level. Adjusted test statistics, taking into account the presence of noise in the data, are presented that overcome this issue. Throughout the chapter, we made a strong parametric assumption on the noise process, namely that it is i.i.d. normal with constant variance and independent of the process generating the efficient price. Relaxing this assumption is left for further research.

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